Summary of materials to be reviewed for the Final Exam (Phy 410)
May 5, 2010
The final exam will be about 75% from Chapters 6-9 and 25% from the earlier chapters.

1. Counting and summing of modes in 3, 2, and 1 dimensions.
$\vec{n} = (n_x, n_y, n_z)$, $\vec{n} = (n_x, n_y)$, $\vec{n} = (n_x)$
2. Momentum $\vec{p} = \hbar\left(\frac{\pi}{L}\right)\vec{n}$.

   Energy

   of photons $\varepsilon_{\vec{p}} = cp$.

   of nonrelativistic particles $\varepsilon_{\vec{p}} = \frac{p^2}{2m}$,

   of relativistic particles $\varepsilon_{\vec{p}} = \sqrt{m^2c^4 + p^2c^2}$

3. Ideal Gas of Fermions and Bosons and their Classical regime

   Fermi Dirac distribution: $f_{FD}(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/\tau} + 1}$

   Bose Einstein distribution: $f_{BE}(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/\tau} - 1}$

   For very low densities $n << n_Q \; OR \; v >> \lambda_{th}^3$

   $f_{FD}(\varepsilon) = f_{BE}(\varepsilon) = e^{-(\varepsilon-\mu)/\tau} = \lambda e^{-\varepsilon/\tau}$

   Chemical potential for classical ideal gas (internal and external)

4. Fermions at $\tau = 0$ (different dimensions), Concept of density of orbitals (or states)

   $\mu(\tau = 0) = \varepsilon_F$; *Fermi energy*

   $f_{FD}(\varepsilon) = 1 \; for \; \varepsilon \le \varepsilon_F$

   $\quad = 0 \; for \; \varepsilon > \varepsilon_F$

   $$N = \int_0^\infty D(\varepsilon) f_{FD}(\varepsilon) d\varepsilon$$

   $$U = \int_0^\infty D(\varepsilon)\varepsilon f_{FD}(\varepsilon) d\varepsilon$$

   In 3,2,1 dimensions

   $$D_3(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon^{1/2}; \; D_2(\varepsilon) = \frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right); \; D_1(\varepsilon) = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1'2}\varepsilon^{-1/2}$$

   $$\text{Pressure}: p = -\left(\frac{\partial U}{\partial V}\right)_N (3d); \; \text{Spreading pressure}: \pi = -\left(\frac{\partial U}{\partial A}\right)_N (2d); \; \text{Tension} = T = -\left(\frac{\partial U}{\partial L}\right)_N (1d)$$

5. Planck Distribution function (photon gas)
Thermal average number of photons and average energy in a single mode of frequency ω and energy $\hbar\omega$ are given by

$$<s>=\frac{1}{e^{\hbar\omega/\tau}-1};<\varepsilon>=\frac{\hbar\omega}{e^{\hbar\omega/\tau}-1}$$

Average energy, Pressure, and Entropy of Black body radiation (photon gas in 3d)

$$\frac{U}{V}=\frac{\pi^2}{15c^3\hbar^3}\tau^4=A\tau^4;p=\frac{1}{3}\frac{U}{V};\ \sigma=\frac{4}{3}VA\tau^3$$

6. Bose Einstein Condensation Temperature(τ_E) is the temperature when the chemical potential becomes zero as one comes down from very high temperature.

$$N=\int_0^\infty D(\varepsilon)\frac{1}{e^{(\varepsilon-\mu)/\tau}-1};\mu\le 0.$$

$$U=\int_0^\infty D(\varepsilon)\,\varepsilon\frac{1}{e^{(\varepsilon-\mu)/\tau}-1};\mu\le 0.$$

7. Heat Capacity and Entropy of Bose gas below the condensation temperature

8. Heat engines (reversible and irreversible), efficiency η; refrigerators and heat pumps, coefficient of performance, COP γ.

9. Work done (on or by) a gas (ideal gas of finite mass particles, photon gas) during isothermal and adiabatic processes.

10. Effective work (chemical or electrical) and its relationship with changes in the Gibbs free energy, $dW'=dG$. Physics/Chemistry of electrolysis and heat pump.

11. Boltzman distribution $p(\varepsilon_s)\propto e^{-\varepsilon_s/\tau}$;

12. Gibb's distribution $p(N,\ \varepsilon_{s(N)})\propto e^{-(\varepsilon_{s(N)}-\mu N)/\tau}=\lambda^N e^{-(\varepsilon_{s(N)})/\tau}$

13. Partition function (when N is fixed) $Z=\sum_s e^{-\varepsilon_s/\tau}$; $F=-\tau\ln Z$

$$<E>=U=\frac{\sum_s \varepsilon_s e^{-\varepsilon_s/\tau}}{\sum_s e^{-\varepsilon_s/\tau}}=\tau^2\left(\frac{\partial\ln Z}{\partial\tau}\right)$$

14. Gibbs sum $\overline{Z} = \sum_{N} \sum_{s(N)} \lambda^{N} e^{-(\varepsilon_{s(N)})/\tau}$;

Calculate <N> from $\overline{Z}; < N >= \lambda \left(\frac{\partial \ln \overline{Z}}{\partial \lambda} \right)_{\tau, V}$

15.

Thermodynamic identities and use it to calculate changes in enthalpy *H* and Helmholtz free energy *F, G*, and other physical quantities.

$$\tau \, d\sigma = dU + p \, dV - \mu \, dN$$

$$H = U + pV; dH = \tau \, d\sigma + V \, dp + \mu \, dN; H(\sigma, p, N)$$

$$F = U - \tau\sigma; dF = -\sigma \, d\tau - p \, dV + \mu \, dN; F(\tau, V, N)$$

$$G = U + pV - \tau\sigma; \; dG = -\sigma \, d\tau + Vdp + \mu \, dN; G(\tau, p, N) = \mu(\tau, p) N$$

$$\textit{The Chemical potential} \quad \mu(\tau, P) = G(\tau, p, N)/N$$

Use these equations to obtain different physical quantities from different free energies under different external (control) constraints.

$$\textit{Entropy at constant } V \textit{ and } N: \; \sigma(\tau, V, N) = -\left(\frac{\partial F}{\partial \tau} \right)_{V, N}$$

$$\textit{Entropy at constant } p \textit{ and } N: \quad \sigma(\tau, p, N) = -\left(\frac{\partial G}{\partial \tau} \right)_{p, N}$$

Maxwell Relations: Know how to derive them:

$$\textit{Use} \quad \left(\frac{\partial}{\partial x} \left(\frac{\partial f(x, y)}{\partial y} \right)_{x} \right)_{y} = \left(\frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial x} \right)_{y} \right)_{x}$$

where $f(x, y)$ *is a Free energy and* x, y *are two thermodynamic variables like* $\tau, \; N, \; p, \; V, \; \sigma, \; N$ *etc*