HW#12

12.1

The maximum possible efficiency should be

$$\eta_{c1} = \frac{T_h - T_l}{T_h} = \frac{480K}{773.15K} \approx 0.6208$$

$$\eta_{c2} = \frac{T_h - T_l}{T_h} = \frac{580K}{873.15K} \approx 0.6643$$

By installing the new material, we can produces electricity

$$\frac{P_2}{P_1} = \frac{\eta_{c2}}{\eta_{c1}} \Rightarrow P_2 = P_1 \times \frac{\eta_{c2}}{\eta_{c1}} = 1.0699 \times 10^9 W$$

$$\Delta P = P_2 - P_1 = 6.99 \times 10^7 W$$

Thus, we can obtain the money we can make a year is

$$\Delta P \times 365 \times 24 \times 0.05 = 3.06 \times 10^{10} dollar = 30.6 million$$

12.2

For the maximum COP, we have

$$\gamma_c = \frac{T_l}{T_h - T_l} = \frac{0.01K}{0.99K} \approx 0.0101$$

Thus

$$\gamma_c = \frac{Q_l}{W} \Rightarrow Q_l = \gamma_c \times W = 10.1J$$

12.3

(a)

For gas of photons, from (4.20), we have

$$U = CV\tau^4, C = \frac{\pi^2}{15\hbar^3 c^3}$$

From (4.23), we have

$$\sigma = \frac{4}{3} CV\tau^3$$

Thus, we can obtain

$$\sigma_3 = \sigma_2 \Rightarrow V_3 {\tau_l}^3 = V_2 {\tau_h}^3 \Rightarrow V_3 = V_2 \frac{{\tau_h}^3}{{\tau_l}^3}$$

$$\sigma_4 = \sigma_1 \Rightarrow V_4 {\tau_l}^3 = V_1 {\tau_h}^3 \Rightarrow V_4 = V_1 \frac{{\tau_h}^3}{{\tau_l}^3}$$

(b)

For the first thermal expansion, we have

$$Q_h = Q_{12} = \tau_h \int_1^2 d\sigma = \tau_h \int_1^2 d\sigma = \tau_h(\sigma_2 - \sigma_1) = \frac{4}{3} C\tau_h{}^4 (V_2 - V_1)$$

$$U_{12} = U_2 - U_1 = C\tau_h{}^4 (V_2 - V_1)$$

Thus

$$W_{12} = U_{12} - Q_{12} = -\frac{1}{3} C\tau_h{}^4 (V_2 - V_1) = -\frac{1}{4} Q_h$$

They are not equal to each other.

(c)

For isentropic stages, we have

$$W_{23} = U_{23} = U_3 - U_2 = C(V_3\tau_l{}^4 - V_2\tau_h{}^4)$$

For $V_3 = V_2(\tau_h{}^3 / \tau_l{}^3)$,

$$W_{23} = C(V_3\tau_l{}^4 - V_2\tau_h{}^4) = -CV_2\tau_h{}^3(\tau_h - \tau_l)$$

Also, we have

$$W_{41} = U_{41} = U_1 - U_4 = C(V_1\tau_h{}^4 - V_4\tau_l{}^4)$$

For $V_4 = V_1(\tau_h{}^3 / \tau_l{}^3)$,

$$W_{41} = C(V_1\tau_h{}^4 - V_4\tau_l{}^4) = CV_1\tau_h{}^3(\tau_h - \tau_l)$$

Thus

$$W_{23} + W_{41} = -C(V_2 - V_1)\tau_h{}^3(\tau_h - \tau_l)$$

They do not cancel each other.

(d)

Similar to (b), we can obtain

$$Q_{34} = \tau_h \int_3^4 d\sigma = \tau_l \int_3^4 d\sigma = \tau_l(\sigma_4 - \sigma_3) = \frac{4}{3} C\tau_l{}^4 (V_4 - V_3)$$

$$U_{34} = U_4 - U_3 = C\tau_l{}^4 (V_4 - V_3)$$

Thus

$$W_{34} = U_{34} - Q_{34} = -\frac{1}{3} C\tau_l{}^4 (V_4 - V_3) = -\frac{1}{3} C\tau_l{}^4 \frac{\tau_h{}^3}{\tau_l{}^3} (V_1 - V_2) = \frac{1}{3} C\tau_l\tau_h{}^3 (V_2 - V_1)$$

Thus, we can sum up

$$W = -(W_{12} + W_{23} + W_{34} + W_{41}) = \frac{4}{3} C(V_2 - V_1)\tau_h{}^3(\tau_h - \tau_l)$$

For $Q_h = \frac{4}{3} C\tau_h{}^4 (V_2 - V_1)$, we can obtain

$$\eta = \frac{W}{Q_h} = \frac{\tau_h - \tau_l}{\tau_h} = \eta_c$$

12.4

From (7.7), (7.10), we have

$$\varepsilon_F = \frac{\hbar^2}{2M}(\frac{3\pi^2 N}{V})^{2/3}$$

$$U_0 = \frac{3}{5}N\varepsilon_F = \frac{3\hbar^2}{10M}(\frac{3\pi^2 N}{V_i})^{2/3} N$$

For apply classical limit, we have

$$U = \frac{3}{2}N\tau_f = \frac{3\hbar^2}{10M}(\frac{3\pi^2 N}{V_i})^{2/3} N$$

$$\Rightarrow \tau_f = \frac{\hbar^2}{5M}(\frac{3\pi^2 N}{V_i})^{2/3}$$

$$\Rightarrow T_f = \frac{\hbar^2}{5Mk_B}(\frac{3\pi^2 N}{V_i})^{2/3}$$

For classical regime, we need

$$n \ll n_Q$$

$$\Rightarrow \frac{N}{V_f} \ll (\frac{M\tau_f}{2\pi\hbar^2})^{3/2} = (\frac{1}{10\pi})^{3/2}\frac{3\pi^2 N}{V_i}$$

$$\Rightarrow \frac{V_f}{V_i} \gg \frac{(10\pi)^{3/2}}{3\pi^2} \approx 5.95$$

The factor the gas should expand is 5.95

(a)

For

$$N/V = 10^{22}cm^{-3} = 10^{28}m^{-3}$$

$$M = 9.11\times 10^{-31}Kg$$

We can obtain

$$T_f = \frac{\hbar^2}{5Mk_B}(\frac{3\pi^2 N}{V_i})^{2/3} \approx 7.85\times 10^3 K$$

(b)

For

$$N/V = 10^{30}cm^{-3} = 10^{36}m^{-3}$$

$$M = 1.67\times 10^{-27}Kg$$

We can obtain

$$T_f = \frac{\hbar^2}{5Mk_B}(\frac{3\pi^2 N}{V_i})^{2/3} \approx 9.23\times 10^5 K$$