HW1 Solutions (Ding Wang – TA for PHY 410, SP10)

1.1

(a)

There are 2^4=16 microstates.

↑↑↑↑, ↑↑↑↓, ↑↑↓↑, ↑↑↓↓, ↑↓↑↑, ↑↓↑↓, ↑↓↓↑, ↑↓↓↓,

↓↑↑↑, ↓↑↑↓, ↓↑↓↑, ↓↑↓↓, ↓↓↑↑, ↓↓↑↓, ↓↓↓↑, ↓↓↓↓

(b)

s=2

↑↑↑↑

Probability = 1/16 =0.0625

s=1

↑↑↑↓, ↑↑↓↑, ↑↓↑↑, ↓↑↑↑

Probability = 4/16 = 0.25

s=0

↑↑↓↓, ↑↓↑↓, ↑↓↓↑, ↓↑↑↓, ↓↑↓↑, ↓↓↑↑

Probability = 6/16 = 0.375

s=-1

↑↓↓↓, ↓↑↓↓, ↓↓↑↓, ↓↓↓↑

Probability = 4/16 = 0.25

s=-2

↓↓↓↓

Probability = 1/16 =0.0625

(c)

$$g(N,s)=\frac{N!}{(\frac{N}{2}+s)!(\frac{N}{2}-s)!}$$

We can get

g(4,2)=1

g(4,1)=4

g(4,0)=6

g(4,-1)=4

g(4,-2)=1

The same result as part (b).

1.2

(a) The number of all possible outcomes is

$Z=2^{20}$ = 1048576

(b) The probability of getting one exact order is

$1/Z = 9.5367\times10^{-7}$

(c) The probability is

$$\frac{C_{20}^{12}}{Z} = \frac{125970}{1048576} = 0.1201$$

1.3

For N=40, we have

$N! = 8.1592\times10^{47}$

Log(N!) = 110.3206

For approximation, we have

$$N! \approx N^N e^{-N}\sqrt{2\pi N} = 8.1422\times10^{47}$$

$$\log N! \approx \frac{1}{2}\log 2\pi + (N+\frac{1}{2})\log N - N = 110.3186$$

1.4

(a)

$$g(N,0) \square (2/\pi N)^{1/2} 2^N = 2.5231\times10^{-12}\times2^{10^{23}}$$

$2^{10^{23}}$ is a huge number which cannot be print out by all the paper of the world.

(b)

The number of microstates explored is

$10^9\times3600\times24\times365\times10\times10^9 = 3.1536\times10^{26}$