HW#2

2.1

From these formulas,

$$\ln N! \approx \frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - N, g(N,s) = \frac{N!}{(\frac{N}{2}+s)!(\frac{N}{2}-s)!}, \ln(1+x) \approx x - \frac{1}{2}x^2 (x << 1)$$

We can get

$$\ln g(N,s) = \ln\frac{N!}{(\frac{N}{2}+s)!(\frac{N}{2}-s)!} = \ln(N!) - \ln[(\frac{N}{2}+s)!] - \ln[(\frac{N}{2}-s)!]$$

$$\approx \frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - N - [\frac{1}{2}\ln 2\pi + (\frac{N}{2}+s+\frac{1}{2})\ln(\frac{N}{2}+s) - (\frac{N}{2}+s)]$$

$$-[\frac{1}{2}\ln 2\pi + (\frac{N}{2}-s+\frac{1}{2})\ln(\frac{N}{2}-s) - (\frac{N}{2}-s)]$$

$$= -\frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - (\frac{N}{2}+s+\frac{1}{2})\ln[\frac{N}{2}(1+\frac{2s}{N})] - (\frac{N}{2}-s+\frac{1}{2})\ln[\frac{N}{2}(1-\frac{2s}{N})]$$

$$\approx -\frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - (\frac{N}{2}+s+\frac{1}{2})[\ln\frac{N}{2} + (\frac{2s}{N} - \frac{2s^2}{N^2})] - (\frac{N}{2}-s+\frac{1}{2})[\ln\frac{N}{2} + (-\frac{2s}{N} - \frac{2s^2}{N^2})]$$

$$= -\frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - (N+1)\ln\frac{N}{2} - \frac{2s^2}{N} + O(N^{-2})$$

$$\approx -\frac{1}{2}(\ln 2 + \ln\pi) + (N\ln N + \frac{1}{2}\ln N) - [(N+1)\ln N - (N+1)\ln 2] - \frac{2s^2}{N}$$

$$= \frac{1}{2}(\ln 2 - \ln\pi - \ln N) + N\ln 2 - \frac{2s^2}{N}$$

$$= \frac{1}{2}\ln(\frac{2}{\pi N}) + N\ln 2 - \frac{2s^2}{N}$$

Thus,

$$g(N,s) \approx e^{\frac{1}{2}\ln(\frac{2}{\pi N}) + N\ln 2 - \frac{2s^2}{N}} = e^{\frac{1}{2}\ln(\frac{2}{\pi N}) + N\ln 2} e^{-\frac{2s^2}{N}} = e^{\ln\sqrt{(\frac{2}{\pi N})}2^N} e^{-\frac{2s^2}{N}} = g(N,0)e^{-\frac{2s^2}{N}}$$

$$g(N,0) = e^{\ln\sqrt{(\frac{2}{\pi N})}2^N}$$

2.2

We need $s_1 + s_2 + s_3 + s_4 = 2$, there are 10 microstates:

(4; 2, 0, 0, 0), (4; 0, 2, 0, 0), (4; 0, 0, 2, 0), (4; 0, 0, 0, 2),
(4; 1, 1, 0, 0), (4; 1, 0, 1, 0), (4; 1, 0, 0, 1), (4; 0, 1, 1, 0), (4; 0, 1, 0, 1), (4; 0, 0, 1, 1)

From the formula,

$$g(N,n) = \frac{(N+n-1)!}{n!(N-1)!}$$

We can get

$$g(4,2)=\frac{5!}{2!3!}=10$$

It is the same result.

2.3

From this formula,

$$g(N,s)=\frac{N!}{(\frac{N}{2}+s)!(\frac{N}{2}-s)!}$$

We can get these results,

$$g(2,1)=1, g(2,0)=2, g(2,-1)=1$$
$$g(4,2)=1, g(4,1)=4, g(4,0)=6, g(4,-1)=4, g(4,-2)=1$$

Check the total system consisting of 4 magnets,

$$N=N_1+N_2, N_1=2, N_2=2, 2s=2s_1+2s_2$$

$$g(4,2)=g_1(2,1)g_2(2,1)=1$$
$$g(4,1)=g_1(2,0)g_2(2,1)+g_1(2,1)g_2(2,0)=4$$
$$g(4,0)=g_1(2,1)g_2(2,-1)+g_1(2,0)g_2(2,0)+g_1(2,-1)g_2(2,1)=6$$
$$g(4,-1)=g_1(2,0)g_2(2,-1)+g_1(2,-1)g_2(2,0)=4$$
$$g(4,-2)=g_1(2,-1)g_2(2,-1)=1$$

We get the same results.

For Stirling's approximation, we need N>>1. For N=4, we cannot use Stirling's approximation.