HW#7

7.1

From (5.12a), (5.60)

$$\mu = \tau \log(n / n_Q), n_Q = (M\tau / 2\pi\hbar^2)^{3/2}$$

$$\lambda = \exp(\mu / \tau) = n / n_Q$$

For Ar gas at 300K, we can get

$$M = 39.95 \times 1.66 \times 10^{-27} kg = 6.63 \times 10^{-26} kg$$

$$n_Q = (M\tau / 2\pi\hbar^2)^{3/2} = (MK_B T / 2\pi\hbar^2)^{3/2} = 2.46 \times 10^{32} m^{-3}$$

(i)

$$n = 10^{16} / cm^3 = 10^{22} m^{-3}$$

$$\mu / \tau = \log(n / n_Q) = -23.93$$

$$\lambda = n / n_Q = 4.06 \times 10^{-11}$$

(ii)

$$n = 10^{18} / cm^3 = 10^{24} m^{-3}$$

$$\mu / \tau = \log(n / n_Q) = -19.32$$

$$\lambda = n / n_Q = 4.06 \times 10^{-9}$$

(iii)

$$n = 10^{16} / cm^3 = 10^{26} m^{-3}$$

$$\mu / \tau = \log(n / n_Q) = -14.72$$

$$\lambda = n / n_Q = 4.06 \times 10^{-7}$$

7.2

Starting from the thermodynamic identity

$$\tau d\sigma = dU + pdV - \mu dN$$

$$\Rightarrow dU - d(\tau\sigma) + \sigma d\tau + pdV - \mu dN = 0$$

$$\Rightarrow d(U - \tau\sigma) + \sigma d\tau + pdV - \mu dN = 0$$

$$\Rightarrow dF + \sigma d\tau + pdV - \mu dN = 0$$

We can get

$$p = \left(\frac{\partial F}{\partial V}\right)_{N,\tau}, \sigma = \left(\frac{\partial F}{\partial \tau}\right)_{N,V}$$

Thus

$$\left(\frac{\partial p}{\partial \tau}\right)_{N,V} = \left(\frac{\partial^2 F}{\partial V \partial \tau}\right)_{N,V,\tau}, \left(\frac{\partial \sigma}{\partial V}\right)_{N,\tau} = \left(\frac{\partial^2 F}{\partial \tau \partial V}\right)_{N,V,\tau}$$

$$\Rightarrow \left(\frac{\partial p}{\partial \tau}\right)_{N,V} = \left(\frac{\partial \sigma}{\partial V}\right)_{N,\tau}$$

7.3

Similar to (5.17), we can get the external potential energy

$$\mu_{ext} = -\int_0^r M\omega^2 r' dr' = -\frac{M\omega^2 r'^2}{2}\Big|_0^r = -\frac{M\omega^2 r^2}{2}$$

Thus

$$\mu = \tau \log(n / n_Q) - \frac{M\omega^2 r^2}{2}$$

In equilibrium, this must be independent of r, thus

$$\tau \log[n(r)/n_Q] - \frac{M\omega^2 r^2}{2} = \tau \log[n(0)/n_Q]$$

$$\Rightarrow \tau \log[n(r)/n(0)] = \frac{M\omega^2 r^2}{2}$$

$$\Rightarrow n(r) = n(0)\exp(\frac{M\omega^2 r^2}{2\tau})$$

7.4

From (5.25)

$$n(B) = n(0)\cosh(mB/\tau)$$

$$\Rightarrow mB/\tau = \cosh^{-1}(\frac{n(B)}{n(0)})$$

$$\Rightarrow m/\tau = \cosh^{-1}(\frac{n(B)}{n(0)})/B$$

From figure (5.6), we can find that

$$\frac{n(B)}{n(0)} = 100, B = 20kG$$

Thus

$$m/\tau = \cosh^{-1}(\frac{n(B)}{n(0)})/B = \cosh^{-1}(100)/20kG \approx 2.6491\times 10^{-4}/G = 2.649/T$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274\times 10^{-24} J/T$$

$$n = m/\mu_B = 2.649\times\tau/\mu_B = 2.649\times k_B T/\mu_B \approx 1183$$