Contemporary Physics

Neutrino mass

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This is a review article about the most recent developments on the field of neutrino mass. The first part of the review introduces the idea of neutrino masses and mixing angles, summarizes the most recent experimental data then discusses the experimental prospects and challenges in this area. The second part of the review discusses the implications of these results for particle physics and cosmology, including the origin of neutrino mass, the see-saw mechanism and sequential dominance, and large extra dimensions and cosmology.

1. Introduction[†]

In 1930, the Austrian physicist Wolfgang Pauli proposed the existence of particles called neutrinos, denoted as v, as a 'desperate remedy' to account for the missing energy in a type of radioactivity called beta decay. At the time physicists were puzzled because nuclear beta decay appeared to violate energy conservation. In beta decay, a neutron in an unstable nucleus transforms into a proton and emits an electron, where the radiated electron was found to have a continuous energy spectrum. This came as a great surprise to many physicists because other types of radioactivity involved gamma rays and alpha particles with discrete energies. Pauli deduced that some of the energy must have been taken away by a new particle emitted in the decay process, the neutrino, which carries energy and has spin 1/2, but which is massless, electrically neutral and very weakly interacting. Because neutrinos interact so weakly with matter, Pauli bet a case of champagne that nobody would ever detect one, and they became known as 'ghost particles'. Indeed it was not until a quarter of a century later, in 1956, that Pauli lost his bet and neutrinos were discovered when Clyde Cowan and Fred Reines detected

antineutrinos emitted from a nuclear reactor at Savannah River in South Carolina, USA.

Since then, after decades of painstaking experimental and theoretical work, neutrinos have become enshrined as an essential part of the accepted quantum description of fundamental particles and forces, the Standard Model of Particle Physics, whose particle content is summarized in figure 1. This is a highly successful theory in which elementary building blocks of matter are divided into three generations of two kinds of particle—quarks and leptons. It also includes three of the fundamental forces of Nature, the strong (g), electromagnetic (γ) and weak (W, Z) forces carried by spin 1 force carrying bosons (shown in parentheses) but does not include gravity. There are six flavours of quarks given in figure 1. The leptons consist of three flavours of charged leptons, the electron e^-, muon μ^- and tau τ^-, together with three flavours of neutrinos—the electron neutrino v_e, muon neutrino v_μ and tau neutrino v_τ which are our main concern here.

The first clues that neutrinos have mass came from an experiment deep underground, carried out by an American scientist Raymond Davis Jr, detecting solar neutrinos [8]. It revealed only about one-third of the number predicted by theories of how the Sun works pioneered by John Bahcall [8]. The result puzzled both solar and neutrino physicists. However, some Russian researchers, Mikheyev and Smirnov, developing ideas proposed previously by

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[†]There are plenty of good reviews of neutrino physics, and here are just a few of them: [1,2,3,4,5,6,7,8]. This Introduction is intended to be a rapid overview of the subject, and much of the material contained here will be explained in greater depth in the body of this review.

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Wolfenstein in the USA, suggested that the solar neutrinos might be changing into something else. Only electron neutrinos are emitted by the Sun and they could be converting into muon and tau neutrinos which were not being detected by the experiments. This effect called 'neutrino oscillations', as the types of neutrino interconvert over time from one kind to another, was first proposed some time earlier by Pontecorvo [9]. The precise mechanism for 'solar neutrino oscillations' proposed by Mikheyev, Smirnov and Wolfenstein involved the resonant enhancement of neutrino oscillations due to matter effects. Just as light passing through matter slows down, which is equivalent to the photon gaining a small effective mass, so neutrinos passing through matter also result in the neutrinos slowing down and gaining a small effective mass. The effective neutrino mass is largest when the matter density is highest, which in the case of solar neutrinos is in the core of the Sun. In particular electron neutrinos generated in the core of the Sun will be subject to such matter effects. It turns out that neutrino oscillations, which would be present in the vacuum due to neutrino mass and mixing, will exhibit strong resonant effects in the presence of matter as the effective mass of the neutrinos varies along the path length of the neutrinos. This can result in a resonant enhancement of solar neutrino oscillations known as the MSW effect [10].

Neutrino oscillations are analogous to coupled pendulums, where oscillations in one pendulum induce oscillations in another pendulum. The coupling strength is defined in terms of something called the 'lepton mixing matrix' $U^{\dagger}$ which relates the basic Standard Model neutrino states, ν_e, ν_μ, ν_τ, associated with the electron, muon and tau, to the neutrino mass states ν_1, ν_2, and ν_3 with mass m_1, m_2, and m_3, as shown in figure 2. According to quantum mechanics it is not necessary that the Standard Model states ν_e, ν_μ, ν_τ be identified in a one–one way with the mass eigenstates ν_1, ν_2, and ν_3, and the matrix elements of U give the quantum amplitude that a particular Standard Model state contains an admixture of a particular mass eigenstate. As with all quantum amplitudes, the matrix elements of U are expected to be complex numbers in general.

The idea of neutrino oscillations gained support from the Japanese experiment Super-Kamiokande [12] which in 1998 showed that there was a deficit of muon neutrinos reaching Earth when cosmic rays strike the upper atmosphere, the so-called 'atmospheric neutrinos'. Since most neutrinos pass through the Earth unhindered, Super-Kamiokande was able to detect muon neutrinos coming from above and below, and found that while the correct number of muon neutrinos came from above, only about a half of the

†The 'lepton mixing matrix' U is also frequently referred to as the Maki–Nakagawa–Sakata (MNS) matrix U_{MNS} [11], and sometimes the name of Pontecorvo is added at the beginning to give U_{PMNS}.
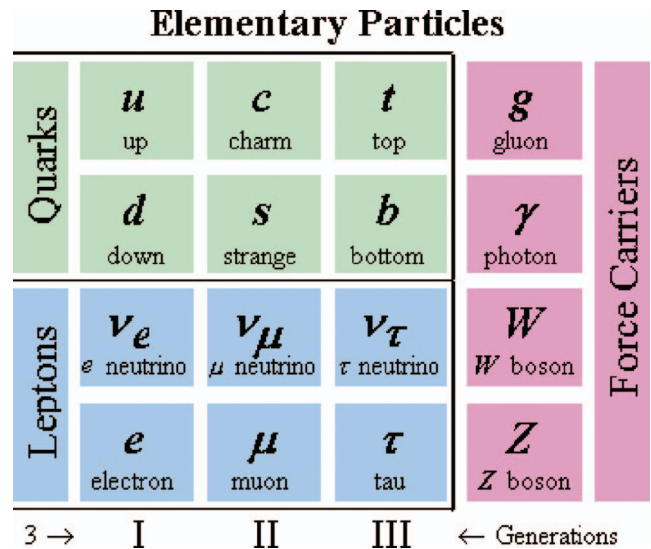

Figure 1. The particles of the Standard Model.
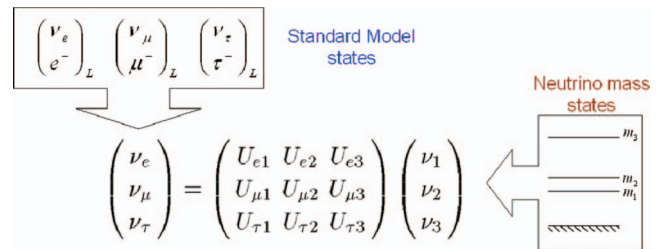
Figure 2. The lepton mixing matrix U relates Standard Model neutrino states ν_e, ν_μ, ν_τ to the neutrino mass states ν_1, ν_2, and ν_3 with mass m_1, m_2, and m_3. The mass states are not necessarily ordered as $m_1 < m_2 < m_3$, as discussed later. The convention is chosen such that ν_1 contains mostly ν_e, while ν_3 contains mainly ν_μ and ν_τ, with very little ν_e. The state ν_2 contains roughly equal amounts of ν_e, ν_μ and ν_τ. The matrix U is unitary, which implies that the probability that each of the states ν_1, ν_2, and ν_3 contains each of ν_e, ν_μ, ν_τ must sum to unity.

expected number came from below. The results were interpreted as half the muon neutrinos from below oscillating into tau neutrinos over an oscillation length L of the diameter of the Earth, with the muon neutrinos from above having a negligible oscillation length, and so not having time to oscillate, yielding the expected number of muon neutrinos from above. More recently, the Sudbury Neutrino Observatory (SNO) in Canada has spectacularly confirmed the 'solar neutrino oscillations' [13]. The experiment measured both the flux of the electron neutrinos and the total flux of all three types of neutrinos. The SNO data revealed that physicists' theories of the Sun were correct

after all, and the solar neutrinos v_e were produced at the standard rate but were oscillating into v_μ and v_τ, with only about a third of the original v_e flux arriving at the Earth. Since then, neutrino oscillations consistent with solar neutrino observations have been seen using man-made neutrinos from nuclear reactors at KamLAND in Japan [14,15], and neutrino oscillations consistent with atmospheric neutrino observations have been seen using neutrino beams fired over hundreds of kilometres as in the K2K experiment in Japan [16], the Fermilab-MINOS experiment in the US [17] or the CERN-OPERA experiment in Europe. Further long-baseline neutrino beam experiments are in the pipeline, and neutrino oscillation physics is poised to enter the precision era, with Superbeams and a Neutrino Factory on the horizon.

Following these results several research groups [18,19] showed that the electron neutrino has a mixing matrix element of $|U_{e2}| \approx 1/3^{1/2}$ which is the quantum amplitude for v_e to contain an admixture of the mass eigenstate v_2 corresponding to a massive neutrino of mass $m_2 \approx 0.007$ electronvolts (eV) or greater (by comparison the electron has a mass of about half a megaelectronvolt (MeV)). The muon and tau neutrinos were observed to contain approximately equal amplitudes of a heavier neutrino v_3 of mass $m_3 \approx 0.05$ eV or greater, $|U_{\mu3}| \approx |U_{\tau3}| \approx 1/2^{1/2}$, where a normalized amplitude of $1/2^{1/2}$ corresponds to a 1/2 fraction of v_3 in each of v_μ and v_τ, leading to a maximal mixing and oscillation of $v_\mu \leftrightarrow v_\tau$. However, according to the results from the CHOOZ nuclear reactor experiment [20], the electron neutrino must only mix very weakly (if at all) with this state, $|U_{e3}| < 0.2$. Neutrino oscillations are only sensitive to mass differences, and the lightest neutrino mass m_1 is not measured, so these mass values are only lower bounds. However, as discussed later, there are cosmological reasons to believe that none of the neutrino masses can exceed about 0.3 eV. Clearly, then, neutrino masses are much smaller than the other charged fermion masses, and this represents something of a puzzle. However there is a more urgent question that must be faced since, unlike the case for quarks and charged leptons, the Standard Model actually predicts that neutrinos have no mass at all!

The most intuitive way to understand why neutrino mass is forbidden in the Standard Model, is to understand that the Standard Model predicts that neutrinos always have a 'left-handed' spin—rather like rifle bullets which spin counter clockwise to the direction of travel. In fact this property was first experimentally measured in 1958, two years after the neutrino was discovered, by Maurice Goldhaber, Lee Grodzins and Andrew Sunyar. More accurately, the 'handedness' of a particle describes the direction of its spin vector along the direction of motion, and the neutrino being 'left-handed' means that its spin vector always points in the opposite direction to its momentum vector. The fact that the neutrino is

left-handed, written as v_L, implies that it must be massless. If the neutrino has mass then, according to special relativity, it can never travel at the speed of light. In principle, a fast moving observer could therefore overtake the spinning massive neutrino and would see it moving in the opposite direction. To the observer, the massive neutrino would therefore appear right-handed. Since the Standard Model predicts that neutrinos must be strictly left-handed, it follows that neutrinos are massless in the Standard Model. It also follows that the discovery of neutrino mass implies new physics Beyond the Standard Model, with profound implications for particle physics and cosmology.

The rest of the review is organized as follows. In section 2 neutrino masses and mixing angles will be defined more precisely, assuming no prior knowledge, and starting with two neutrino mixing, building up to three neutrino mixing, eventually with complex CP violating phases. Also the the current experimental status and future prospects will be discussed in some more detail. In section 3 the implications of neutrino mass for particle physics and cosmology are described, including the origin of neutrino mass, the seesaw mechanism and sequential dominance, and large extra dimensions and cosmology. Finally section 4 concludes the review.

2. Neutrino masses and mixing angles

The history of neutrino oscillations dates back to the work of Pontecorvo who in 1957 [9] proposed $v \to \bar{v}$ oscillations in analogy with $K \to \bar{K}$ oscillations, described as the mixing of two Majorana neutrinos. Majorana neutrinos will be explained later in this review, but for now it is sufficient to define them as neutrinos which are equivalent to their own antiparticles. Pontecorvo was the first to realize that what we call the 'electron neutrino', for example, may be a linear combination of mass eigenstate neutrinos, and that this feature could lead to neutrino oscillations of the kind $v_e \to v_\mu$. Later on MSW proposed that such neutrino oscillations could be resonantly enhanced in the Sun [10]. The present section introduces the basic formalism of neutrino masses and mixing angles, gives an up-to-date summary of the current experimental status of this fast moving field, and discusses future experimental prospects.

2.1 *Two state atmospheric neutrino mixing*

In 1998 the Super-Kamiokande experiment published a paper [12] which represents a watershed in the history of neutrino physics. The Super-Kamiokande experiment consists of thousands of tonnes of pure water in a tank deep underground, and was originally built to search for proton decay. However, its designers realized that the experiment might also be able to detect highly energetic neutrinos from the Sun that interact with electrons via

scattering reactions. These electrons can travel faster than the local speed of light in the water, causing them to emit the optical equivalent of a sonic boom—a glow of blue light called Cerenkov radiation that can be detected by ultra-sensitive photomultiplier tubes around the tank. Super-Kamiokande also measured the number of electron and muon neutrinos that arrive at the Earth's surface as a result of cosmic ray interactions in the upper atmosphere, which are referred to as 'atmospheric neutrinos'. While the number and angular distribution of electron neutrinos is as expected, Super-Kamiokande showed that the number of muon neutrinos is significantly smaller than expected and that the flux of muon neutrinos exhibits a strong dependence on the zenith angle. These observations gave compelling evidence that muon neutrinos undergo flavour oscillations and this in turn implies that at least one neutrino flavour has a non-zero mass. The standard interpretation, well supported by current data, is that muon neutrinos are oscillating into tau neutrinos.

Current atmospheric neutrino oscillation data are well described by simple two-state mixing

$$\begin{pmatrix} v_\mu \\ v_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} v_2 \\ v_3 \end{pmatrix}, \qquad (1)$$

and the two-state probability oscillation formula

$$P(v_\mu \to v_\tau) = \sin^2 2\theta_{23} \, \sin^2\left(1.27\Delta m_{32}^2 L/E\right), \qquad (2)$$

where

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \qquad (3)$$

and m_i are the physical neutrino mass eigenvalues associated with the mass eigenstates v_i. Δm_{32}^2 is in units of eV^2, the baseline L is in km and the beam energy E is in GeV. The atmospheric data results support maximal mixing, with best-fit two-neutrino oscillation parameters of

$$\sin^2 2\theta_{23} = 1, \quad \Delta m_{32}^2 = 2.6 \times 10^{-3}\,eV^2. \qquad (4)$$

The 90% C.L. range for Δm_{32}^2 at $\sin^2 2\theta_{23} = 1$ is between 2.0 and 3.2×10^{-3} eV^2. The experimental results for such neutrino oscillations are usually plotted as confidence level contours in the $\Delta m_{32}^2 - \sin^2 2\theta_{23}$ plane as shown in figure 3. The results are dominated by the latest Super-Kamiokande results, but the recent results from the long baseline neutrino beam experiments K2K [16] and MINOS [17] are also shown on the same plot.

The approximately maximal mixing angle $\theta_{23} = 45°$ means that we identify the heavy atmospheric neutrino of mass m_3 as being approximately

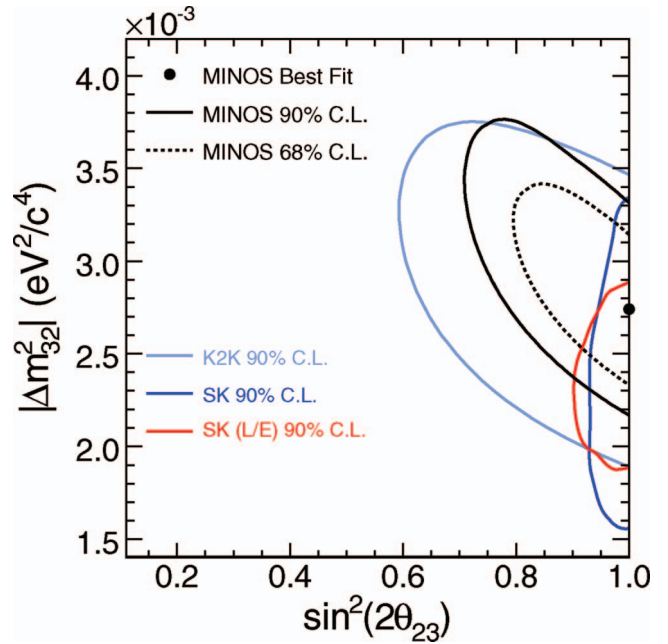$$v_3 \approx \frac{v_\mu + v_\tau}{2^{1/2}} \qquad (5)$$

Figure 3. Confidence intervals from the MINOS experiment [17]. Results from K2K and Super-Kamiokande (SK) are also shown. Reprinted figure 4 with permission from D.G. Michael, *et al.*, Phys. Rev. Lett. **97** 191801 (2006). Copyright 2006 by the American Physical Society.

and in addition there is a lighter orthogonal combination of mass m_2, where $m_3^2 - m_2^2 = 2.6 \times 10^{-3}$eV^2. If $m_3 \gg m_2$ then this implies $m_3 \approx 0.05$ eV.

2.2 Three family neutrino mixing

Super-Kamiokande is also sensitive to the electron neutrinos arriving from the Sun, the 'solar neutrinos', and has independently confirmed the reported deficit of such solar neutrinos long reported by other experiments. For example Davis's Homestake Chlorine experiment which began data taking in 1970 consists of 615 tons of tetrachloroethylene, and uses radiochemical techniques to determine the Ar^{37} production rate [8]. More recently the SAGE and Gallex experiments contain large amounts of Ga^{71} which is converted to Ge71 by low energy electron neutrinos arising from the dominant pp reaction in the Sun [8]. The combined data from these and other experiments implies an energy dependent suppression of solar neutrinos which can be interpreted as due to flavour oscillations. Taken together with the atmospheric data, this requires that a second neutrino flavour has a non-zero mass.

SNO is a water Cerenkov detector like Super-Kamiokande, but instead of using normal water it uses heavy water, D$_2$O. The deuterons, D, in the heavy water are the most weakly bound of all nuclei, which gives SNO the

chance to observe three different reactions induced by solar neutrinos. The first of these processes is the charged-current (CC) reaction $v_e + D \rightarrow p + p + e^-$, which is detected by observing Cerenkov photons from the energetic recoil electron, e^-. SNO also measures the neutral-current (NC) reaction $v_\alpha + D \rightarrow p + n + v_\alpha$. This is observed via the emitted neutrons, n, and is independent of the flavour of the incoming neutrino, v_α. It therefore provides a way to normalize the total flux of neutrinos being emitted by the Sun. Finally SNO measures the elastic scattering (ES) reaction also measured in Super-Kamiokande, $v_\alpha + e^- \rightarrow v_\alpha + e^-$, which has some sensitivity to all neutrino flavours. SNO measurements of CC reaction on deuterium is sensitive exclusively to v_e's, while the ES of electrons also has a small sensitivity to v_μ's and v_τ's. The CC ratio is significantly smaller than the ES ratio. This immediately disfavours oscillations of $v'_e s$ to sterile neutrinos[†] which would lead to a diminished flux of electron neutrinos, but equal CC and ES ratios. On the other hand, the different ratios are consistent with oscillations of v_e's to active neutrinos v_μ's and v_τ's since this would lead to a larger ES rate since this has a neutral current component. The SNO analysis is nicely consistent with both the hypothesis that electron neutrinos from the Sun oscillate into other active flavours, and with the Standard Solar Model prediction. The latest results from SNO including the data taken with salt inserted into the detector to boost the efficiency of detecting the neutral current events [13], strongly favour the large solar mixing angle (LMA) MSW solution, discussed more below. In other words there is no longer any solar neutrino problem: we have instead solar neutrino mass!

The minimal neutrino sector required to account for the atmospheric and solar neutrino oscillation data thus consists of three light physical neutrinos with left-handed flavour eigenstates, v_e, v_μ, and v_τ, defined to be those states that share the same doublet as the charged lepton mass eigenstates e, μ, τ (see figure 2). Within the framework of three-neutrino oscillations, the neutrino flavour eigenstates v_e, v_μ, and v_τ are related to the neutrino mass eigenstates v_1, v_2, and v_3 with mass m_1, m_2, and m_3, respectively, by a 3×3 unitary matrix called the lepton mixing matrix U [11]

$$\begin{pmatrix} v_e \\ v_\mu \\ v_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}. \quad (6)$$

If the light neutrinos are Majorana, U can be parameterized in terms of three mixing angles θ_{ij} and three complex phases. A unitary matrix has six phases but three of them are removed by the phase symmetry of the charged lepton Dirac masses. Since the neutrino masses are Majorana

†Sterile neutrinos are defined to be a light neutrino with no weak interactions.

there is no additional phase symmetry associated with them, unlike the case of quark mixing where a further two phases may be removed.

If we begin by assuming that the phases are zero, then the lepton mixing matrix may be parameterized by a product of three Euler rotations, as depicted in figure 4, and given by a product of three matrices:

$$U = R_{23} R_{13} R_{12}, \quad (7)$$

where

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \quad R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix},$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. Note that the allowed range of the angles is $0 \leq \theta_{ij} \leq \pi/2$. Including phases, the lepton mixing matrix is summarized in figure 5. The phases $\alpha_{1,2}$ are called Majorana phases since they are only present if the neutrino mass is Majorana (as defined earlier and discussed later). The phase δ is called the Dirac phase since it is always present even if neutrinos have Dirac mass. We have already seen that the first matrix in figure 5 is associated with atmospheric neutrino oscillations. We now discuss the physics associated with the other matrix factors.

The physics of the second matrix in figure 5 is associated with reactor neutrino oscillations. Reactor experiments detect the anti-electron neutrinos which are produced copiously in the cores of nuclear reactors, and interpret any deficit in the expected number of such particles in terms of neutrino oscillations. The solar neutrino background is
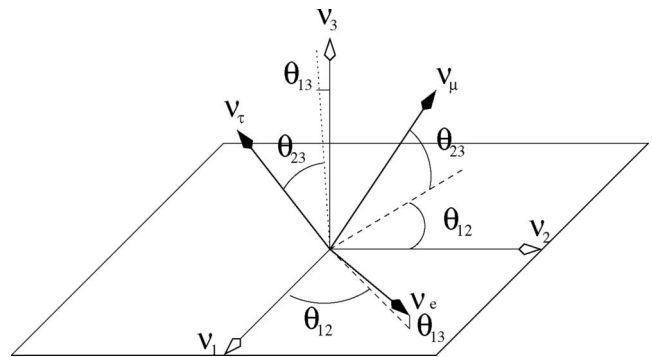
Figure 4. The relation between the neutrino weak eigenstates v_e, v_μ, and v_τ and the neutrino mass eigenstates v_1, v_2, and v_3 in terms of the three mixing angles θ_{12}, θ_{13}, θ_{23}. Ignoring phases, these are just the Euler angles respresenting the rotation of one orthogonal basis into another.

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric	**Reactor**	**Solar**	**Majorana**

Figure 5. The lepton mixing matrix with phases factorizes into a matrix product of four matrices, associated with the physics of atmospheric neutrino oscillations, reactor neutrino oscillations, solar neutrino oscillations and a Majorana phase matrix.

low because the Sun produces electron neutrinos, with negligible numbers of anti-electron neutrinos. The CHOOZ reactor experiment in France failed to see any signal of anti-neutrino oscillations over the Super-Kamiokande mass range. CHOOZ data from $\bar{\nu}_e \to \bar{\nu}_e$ disappearance not being observed provides a significant constraint on θ_{13} over the Super-Kamiokande (SK) prefered range of Δm_{32}^2 [20]:

$$\sin^2\theta_{13} < 0.04. \tag{9}$$

The CHOOZ experiment therefore limits $\sin\theta_{13} \sim 0.2$ or $\theta_{13} \sim 12°$ over the favoured atmospheric range at 90% C.L. The experiment is currently being upgraded to Double CHOOZ, to increase the sensitivity on the angle θ_{13}. The phase δ also appears in the third matrix, and physically represents CP violation (see section 3.1 for a discussion of CP violation). Since the angle θ_{13} that it is associated with has not yet been measured, it might seem somewhat premature to discuss the phases associated with this angle. Nevertheless, there is in fact a huge experimental effort under way to both measure the angle θ_{13} and the CP phase δ. However, it should be emphasized that the CP-violation in the lepton sector is one of the most challenging frontiers in the future studies of neutrino mixing. Nevertheless, the experimental searches for CP-violation in neutrino oscillations can help answer the fundamental question about the status of CP-symmetry in the lepton sector at low energy. The observation of leptonic CP-violation at low energies will have far reaching consequences, and can shed light, in particular, on the possible origin of the baryon asymmetry of Universe.

The physics of the third matrix in figure 5 is associated with Solar neutrino oscillations, as discussed above, and recently confirmed by the Japanese reactor experiment KamLAND, that measures $\bar{\nu}_e$'s produced by several surrounding nuclear reactors [15]. KamLAND has already seen a signal of neutrino oscillations over the Solar neutrino LMA MSW mass range, and has recently confirmed the LMA MSW region 'in the laboratory' [14]. KamLAND and SNO results when combined with other solar neutrino data especially that of Super-Kamiokande uniquely specify the large mixing angle (LMA) MSW [10]

solar solution with three active light neutrino states, a large solar angle

$$\sin^2\theta_{12} \approx 0.30, \quad \Delta m_{21}^2 \approx 7.9 \times 10^{-5} \text{eV}^2, \tag{10}$$

according to the most recent global fits [19]. KamLAND has thus not only confirmed solar neutrino oscillations, but has also uniquely specified the large mixing angle (LMA) solar solution, heralding a new era of precision neutrino physics.

The physics of the fourth matrix in figure 5 is associated with Majorana neutrino masses. These phases could in principle be measured in neutrinoless double beta decay, discussed later.

It is clear that neutrino oscillations, which only depend on $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, give no information about the absolute value of the neutrino mass squared eigenvalues m_i^2, and there are basically two patterns of neutrino mass squared orderings consistent with the atmospheric and solar data as shown in figure 6. Three family oscillation probabilities depend upon the time-of-flight (and hence the baseline L), the Δm_{ij}^2, and U (and hence θ_{12}, θ_{23}, θ_{13}, and δ).

In summary, evidence for neutrino oscillations comes from a wide variety of sources, and the current status of all neutrino oscillation experiments is summarized in figure 7. Though this figure is rather busy, the allowed atmospheric region can be identified by its high value of $\Delta m^2 \approx 3 \times 10^{-3}$ eV^2, corresponding to the region labelled 'SuperK 90/99%' in figure 7. The allowed solar region can be located from its value of $\Delta m^2 \approx 8 \times 10^{-5}$ eV^2, corresponding to the intersection of the upper SNO kidney shaped region with the thin upper KamLAND region in figure 7. These allowed atmospheric and solar regions are plotted again in figure 8, and correspond to the values summarized in table 1 [19].

2.3 Tri-bimaximal mixing

The current experimental situation for neutrino mixing can be summarized by $\sin^2\theta_{23} = 0.5 \pm 0.1$, $\sin^2\theta_{12} = 0.30 \pm 0.03$, $\sin^2\theta_{13} < 0.04$. Maximal mixing corresponds to $\sin^2\theta_{23} = 1/2$, and to first order in the small
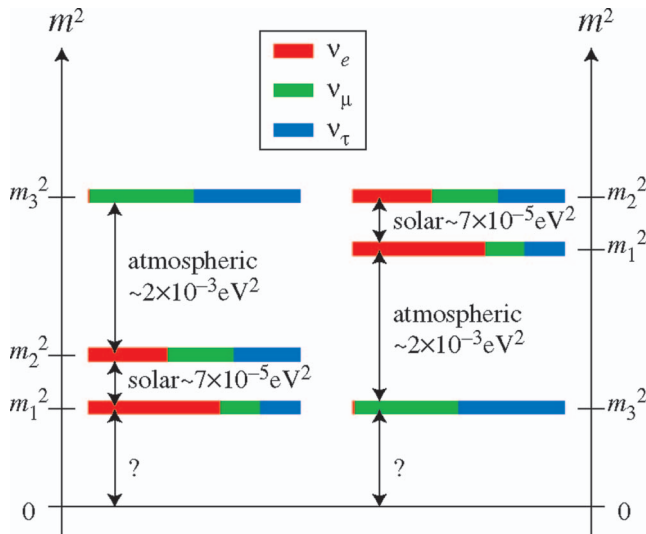
Figure 6. Alternative neutrino mass patterns that are consistent with neutrino oscillation explanations of the atmospheric and solar data. The pattern on the left (right) is called the normal (inverted) pattern. The coloured bands represent the probability of finding a particular weak eigenstate v_e, v_μ, and v_τ in a particular mass eigenstate. The absolute scale of neutrino masses is not fixed by oscillation data and the lightest neutrino mass may vary from 0.0–0.3 eV.

Figure 7. Summary of the currently allowed regions from a global analysis of atmospheric and solar neutrino experiments including first results from KamLAND (taken from H. Murayama's web site http://hitoshi.berkeley.edu/neutrino/.) Reprinted with permission from H. Murayama.

reactor angle the lepton mixing matrix can then be written as:

$$U \approx \begin{pmatrix} c_{12} & s_{12} & \theta_{13} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{11}$$

Tri-bimaximal lepton mixing [21]: corresponds to the choice: $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{13} = 0$,

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{12}$$

In terms of the coloured bands in figure 6 tri-bimaximal mixing corresponds to the following: the state v_3 with mass m_3 consists of a half and half mixture of v_μ and v_τ; the state v_2 with mass m_2 is made up of equal thirds of v_e, v_μ and v_τ; and the state v_1 with mass m_1 comprises two thirds v_e, a sixth v_μ and a sixth v_τ.

   Assuming tri-bimaximal mixing there is a very simple interpretation of atmospheric and solar neutrino oscillations. The interpretation of atmospheric oscillations is that the muon neutrino v_μ contains a large admixture $|U_{\mu3}| = 1/2^{1/2}$ of the third mass eigenstate $v_3 = (v_\mu + v_\tau)/2^{1/2}$, giving
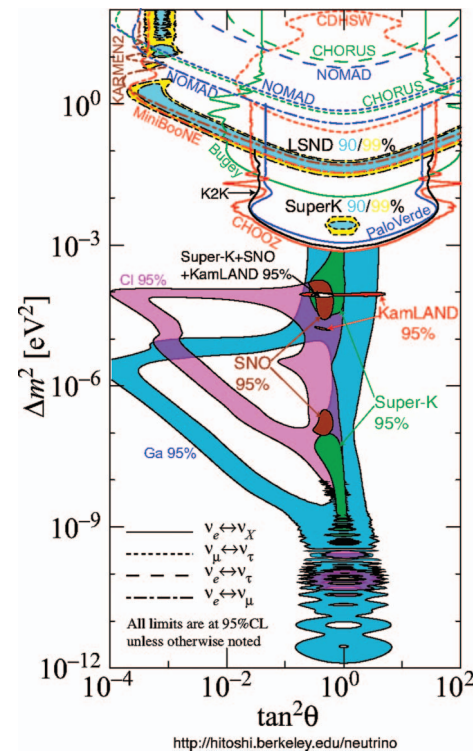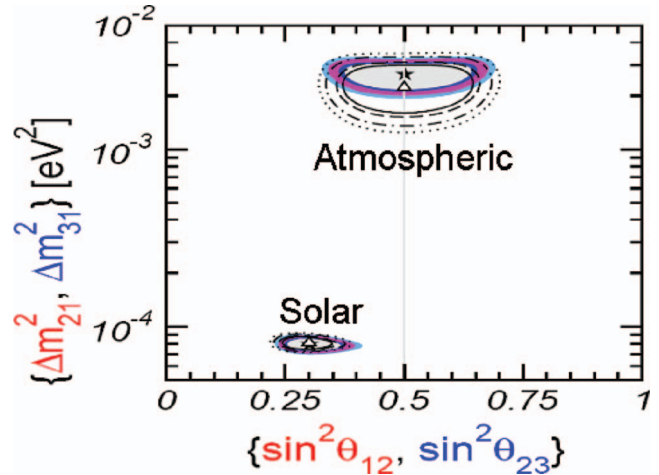

Figure 8. Summary of the currently allowed regions from a global analysis of atmospheric and solar neutrino experiments, taken from [19] where details concerning these plots may be found. Reprinted figure 12 with permission from M. Maltoni, T. Schwetz, M. Tórtola and J.W.F. Valle, *New J. Phys.* **6** 122 (2004). Copyright 2004 by the Institute of Physics.

Table 1. Best-fit values, 2σ, 3σ and 4σ intervals (1 dof) for the three-flavour neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments, taken from [19].

Parameter	Best fit	2σ	3σ	4σ
$\Delta m_{21}^2 [10^{-5}\text{eV}^2]$	7.9	$7.3 - 8.5$	$7.1 - 8.9$	$6.8 - 9.3$
$\Delta m_{31}^2 [10^{-3}\text{eV}^2]$	2.6	$2.2 - 3.0$	$2.0 - 3.2$	$1.8 - 3.5$
$\sin^2 \theta_{12}$	0.30	$0.26 - 0.36$	$0.24 - 0.40$	$0.22 - 0.44$
$\sin^2 \theta_{23}$	0.50	$0.38 - 0.63$	$0.34 - 0.68$	$0.31 - 0.71$
$\sin^2 \theta_{13}$	0.000	≤ 0.025	≤ 0.040	≤ 0.058

a maximal mixing of v_μ with v_τ, with an average probability of $1/2$ of finding a v_τ in an initial pure v_μ beam. The interpretation of solar oscillations is that the electron neutrino v_e contains a large admixture $|U_{e2}| = 1/3^{1/2}$ of a second mass eigenstate $v_2 = (v_e + v_\mu - v_\tau)/3^{1/2}$, giving tri-maximal mixing of v_e with v_μ and v_τ, with an average probability of $1/3$ each of finding a v_μ or a v_τ in an initial v_e beam, and an average probability of $1/3$ that the v_e remains a v_e[†].

Given the one sigma experimental errors above, there is no good reason to believe that lepton mixing takes the tri-bimaximal form exactly. However, it clearly is consistent with the data, at worst gives a nice mnemonic for the lepton mixing matrix, and at best can provide some clues for the construction of a theory of neutrino mixing.

2.4 The LSND signal

The signal of another independent mass splitting from the LSND (Liquid Scintilator Neutrino Detector) accelerator experiment [22]. The LSND collaboration found an excess of electron antineutrinos from a beam of neutrinos consisting of the decay products of a pion particle beam at the Los Alamos Meson Physics Facility (LAMPF) accelerator in New Mexico. The conclusion was that muon antineutrinos in the beam were changing into electron antineutrinos while propagating. This would either require a further light neutrino state with no weak interactions (a so-called 'sterile neutrino') or some other non-standard physics. This effect has not been confirmed by a similar experiment KARMEN [23], and a decisive experiment MiniBooNE has recently reported its first results [24]. In figure 7 the LSND signal region is indicated, together with the KARMEN and MiniBooNE excluded regions. In particular MiniBooNE excludes the simplest two neutrino oscillation interpretation of the LSND signal at 98% C.L.

[24]. Indeed there seems to be no particular motivation for including light sterile neutrinos at the present time coming from theory, experiment or cosmology.

2.5 Experimental prospects and challenges

Neutrino physics has, now entered the precision era. Future neutrino oscillation experiments, will give accurate information about the mass squared splittings $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, mixing angles, and the CP violating phase δ. Long-baseline neutrino beam experiments will given more accurate determinations of the atmospheric parameters, eventually to 10%.

The MINOS (Main Injector Neutrino Oscillation Search) experiment was also proposed in 1995, with a neutrino beam pointed from Fermilab to the Soudan mine in Minnesota, with a baseline of 735 km. The experiment started running in the spring of 2005, and within a year had gathered data corresponding to 1.27×10^{20} protons on target. The first results from MINOS [17] were shown in figure 3. MINOS will run for 5 years, with a goal of accumulating 16×10^{20} protons on target, which should improve our knowledge of the oscillation parameters dramatically. In addition a neutrino beam from CERN to the OPERA detector in the Gran Sasso tunnel is presently underway, and experimenters are looking for τ tracks to prove conclusively that muon neutrinos oscillate to tau neutrinos.

In the next couple of years T2K [25], a Japanese experiment sending a neutrino beam from the J-PARC complex to Super-Kamiokande is due to start. It will be an 'off-axis superbeam' over a baseline of 295 km. Neutrino beams originate from charged pion decays, which generally results in a large spread of neutrino energies. However, for a specific angle relative to the pion direction, the neutrinos have a quite monochromatic energy spectrum and therefore such 'off-axis' neutrino beams will have quite a well-defined energy, which can be advantageous for certain experimental measurements. Its first goal is to measure θ_{13} or set a limit on it of about 0.05 (as compared to the CHOOZ limit on θ_{13} of about 0.2). Interestingly MINOS over a LBL of 735 km is more sensitive than J-PARC to matter effects, so there should be some interesting complementarity between these two experiments, which could for example allow the sign of Δm_{32}^2 to be determined. An 'off-axis superbeam' version of the MINOS experiment called NOνA [26] is seeking approval in the US.

The ultimate goal of oscillation experiments, however, is to measure the CP violating phase δ. To do this it would seem that all the stops would need to be pulled out in neutrino physics experiments. Various Superbeam, or Beta-beam or Neutrino Factory options are currently being considered. For example an upgraded J-PARC with a 4 MW proton driver and a 1 megaton Hyper-Kamiokande detector [25], or some sort of Neutrino Factory based on

[†]The small admixture $|U_{e3}| < 0.2$ of the third mass eigenstate $v_3 \approx (v_\mu + v_\tau)/2^{1/2}$ does not play an important role in solar neutrino oscillations.

muon storage rings would seem to be required for this purpose [27].

However, oscillation experiments are not capable of telling us anything about the absolute scale of neutrino masses. Tritium beta decay end point experiments measure the 'electron neutrino mass' defined by

$$m_{\nu_e} \equiv \left(\sum_i |U_{ei}|^2 m_i^2 \right)^{1/2}. \tag{13}$$

The present Mainz limit is 2.2 eV [28]. The forthcoming KATRIN [29] experiment has a proposed sensitivity of 0.35 eV.

Establishing whether the neutrinos with definite mass ν_j are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e. spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the origin of ν-masses and mixing and the underlying symmetries of particle interactions. The only experiments which have the potential of establishing the Majorana nature of massive neutrinos are the neutrinoless double beta-decay experiments searching for the nuclear decay process $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$, where A is the number of protons plus neutrons and Z is the number of protons in the decaying nucleus (for a review see e.g. [30,31]). Neutrinoless double beta is only sensitive to Majorana masses and effectively measures the combination

$$\langle m_{\beta\beta_{0\nu}} \rangle \equiv \left| \sum_i |U_{ei}|^2 m_i e^{i\alpha_i} \right|. \tag{14}$$

Note the appearance of the Majorana phases α_i from the fourth matrix in figure 5. These phases can lead to cancellations in the sum over the mass flavours, where a precise cancellation would correspond to a Dirac mass. Such experiments are very important since they would not only establish the neutrino mass scale, but would also establish the nature of the neutrino mass, since the process is only allowed if neutrinos have Majorana mass. There has been a recent claim of a signal in neutrinoless double beta decay corresponding to $\langle m_{\beta\beta_{0\nu}} \rangle \approx 0.4$ eV in an analysis of the Heidelberg–Moscow ^{76}Ge experiment [32]. However, this claim has been criticized by two groups [33,34] and in turn this criticism has been refuted [35], followed by a further paper containing a more refined analysis [36]. This claim will be directly tested in the near future by other forthcoming ^{76}Ge experiments such as Majorana and GERDA which will achieve sensities of about $\langle m_{\beta\beta_{0\nu}} \rangle \approx 0.05 - 0.1$ eV [31]. Similar sensitivities are also planned in different isotopes by other forthcoming experiments such as CUORE, Super-NEMO, COBRA, EXO [31] or SNO++, the recent exciting proposal to fill the now decommissioned SNO vessel with liquid scintillator doped with neodymium [37]. The most ambitious sensitivities planned are down to $\langle m_{\beta\beta_{0\nu}} \rangle \approx 0.01$ eV [31].

Let us end this section by summarizing the main experimental challenges facing neutrino physics at the present time. The following challenges can be addressed by future neutrino oscillation experiments.

- The sign of Δm_{31}^2: whether the neutrino mass ordering is 'normal' or 'inverted' has important implications for Grand Unification, Flavour Models and Cosmology.
- The question of CP-violation (δ): measurement of the oscillation phase represents the Holy Grail of neutrino physics, since it would signal CP violation in the lepton sector, which would also have profound implications for Grand Unification, Flavour Models and Cosmology.
- High precision measurements of mixing angles: especially θ_{13} which has so far not been measured at all; a high precision determination of all the mixing angles again provides crucial information for Grand Unification and Flavour Models.

The remaining challenges can be addressed by a combination of neutrinoless double beta decay experiments, tritium end-point experiments and cosmological considerations:

- Majorana versus Dirac: the question of whether neutrino masses are Majorana or Dirac in nature has profound implications for particle physics.
- The absolute neutrino mass scale: only mass squared differences are relevant for neutrino oscillations, and the absolute neutrino mass scale is so far not measured.

3. Implications for particle physics and cosmology

In this section we discuss the origin and nature of neutrino mass, and emphasize that, whatever its origin, it must correspond to new physics Beyond the Standard Model. We then discuss the see-saw mechanism, which is a natural and appealing explanation of small neutrino masses, and its application to atmospheric and solar oscillation data using the sequential dominance mechanism. We also discuss an alternative explanation of small neutrino masses in terms of extra space dimensions. Finally we discuss some cosmological implications of neutrino mass.

3.1 *The origin of neutrino mass*

Neutrino mass is zero in the Standard Model for three independent reasons:

(1)   there are no right-handed neutrinos ν_R;
(2)   there are only Higgs doublets (H^+, H^0);
(3)   the theory is renormalizable.

In the SM these conditions all apply and so neutrinos are massless with ν_e, ν_μ, ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ. Neutrinos and antineutrinos are distinguished by the total conserved lepton number $L = L_e + L_\mu + L_\tau$. To generate neutrino mass we must relax one or more of these conditions. For example, by adding right-handed neutrinos the Higgs mechanism of the Standard Model can give neutrinos the same type of mass as the electron mass or other charged lepton and quark masses.

We begin by discussing the Higgs mechanism of the Standard Model. The Higgs mechanism, originally proposed by the British physicist Peter Higgs, is the mechanism that gives mass to all elementary particles in particle physics. It makes the W boson different from the photon, for example. It can be understood as an elementary case of tachyon condensation where the role of the tachyon is played by a scalar field called the Higgs field. The massive quantum excitation of the Higgs field is also called the Higgs boson. According to the Standard Model all of space is filled by a background Higgs field, which is somewhat analogous to the background electric and magnetic fields that are also present in deep space. In the Standard Model the background Higgs field is due to a single doublet consisting of one charged and one neutral Higgs field (H^+, H^0), where only the neutral field H^0 is switched on in the vacuum, breaking the symmetry of the doublet, and hence breaking the symmetry between the weak and the electromagnetic interactions, resulting in W, Z masses. It also results in fermion masses due to their interaction with the background Higgs field. As an electron travels through space it is continually interacting with the background Higgs field as illustrated in the upper diagram in figure 9, resulting in its mass. However, with each interaction its handedness changes, so that its mass can be thought of as an interaction between a left-handed electron e_L^- and a right-handed electron e_R^- as shown in figure 10. Such an interaction gives rise to what is known as a Dirac mass, named after Paul Dirac, an English physicist who proposed the equation describing massive electrons that bears his name. Strictly speaking such mass terms appear in the Lagrangian density for the quantum field theory, but from our point of view here they may simply be regarded as interactions between a left-handed electron and a right-handed electron, and no knowledge of quantum field theory is required to understand this basic point.

It is possible to add right-handed neutrinos ν_R to the Standard Model, providing that the right-handed neutrinos do not take part in the weak interaction so as to not contradict with the result of Goldhaber *et al.* that weakly interacting neutrinos are always left-handed. With right-handed neutrinos present a similar interaction can take place as for electrons, giving rise to a Dirac mass for the neutrino m_{LR}, as shown in the centre diagram in figure 9
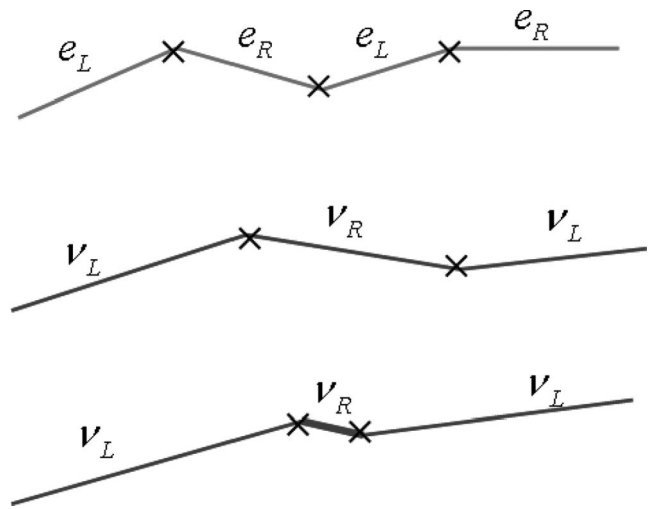

Figure 9. A diagramatic illustration of fermion masses in the presence of a background Higgs field H^0 which is uniformly switched on in the vacuum. In the upper diagram, a left-handed electron mass interacts with the background Higgs field to become a right-handed electron, then interacts again to become a left-handed electron, and so on, resulting in a Dirac mass m_e for the electron. In the centre diagram a similar thing can happen to the neutrino provided right-handed neutrinos are introduced into the Standard Model, leading to a Dirac neutrino mass m_{LR}. The lower diagram shows what happens when the right-handed neutrino acquires a large mass M_{RR} independently of the Higgs mechanism. In this case, the heavy right-handed neutrino cannot travel very far due to its large mass, and in the limit of extremely large M_{RR}, when the length of its propagation goes to zero, the lower diagram looks effectively like a direct interaction between two left-handed neutrinos, resulting in an effective left-handed Majorana mass $m_{LL}^{eff} = m_{LR}^2/M_{RR}$. This is called the see-saw mechanism.

and the upper part of the diagram in figure 11. In principle it is also possible to give neutrinos a new kind of mass called a Majorana mass m_{LL}, named after the Sicilian physicist, Ettore Majorana, if the left-handed neutrino ν_L interacts with its own charge and parity conjugated state, the right-handed antineutrino ν_L^c, where the superscript c denotes the simultaneous operation of charge conjugation (C) (replacing the particle by the antiparticle) and parity (P) (replacing the particle by its mirror image, which has the effect of reversing the spin direction). Such a Majorana mass m_{LL} is shown in the lower part of figure 11. In principle right-handed neutrinos ν_R can also independently acquire their own Majorana masses M_{RR}, by interacting with their own CP conjugates ν_R^c as shown in figure 12. Such Majorana masses m_{LL} or M_{RR} are only possible in principle for neutrinos since they are the only leptons which

Figure 10. The electron Dirac mass m_e can be thought of as an interaction between a left-handed electron e_L^- and a right-handed electron e_R^-. In these figures the long (blue) arrows denote the electron momentum vector, and the short (red) arrows denote the electron spin vector. For right-handed electrons e_R^- the spin vector and the momentun vector are aligned, whereas for left-handed electrons e_L^- they are anti-aligned. Such mass terms $m_e \bar{e}_L^- e_R^-$ appear in the Lagrangian density for the quantum field theory, where the bar over the e_L^- has a conventional meaning that need not concern us here. From our point of view here such mass terms may simply be regarded as interactions that enable left-handed electrons to interact with right-handed electrons.
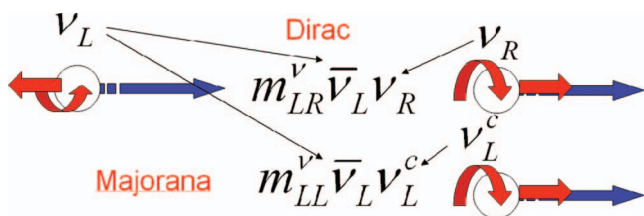

Figure 11. For neutrinos there are two types of mass that are possible. As in the case of the electron there is the Dirac mass m_{LR}^v that couples a left-handed neutrino v_L to a right-handed neutrino v_R, as shown in the upper part of the diagram. However, the role of a right-handed neutrino can be played by v_L^c obtained by transforming the left-handed neutrino v_L under the operations charge and parity conjugatation, where v_L^c is a right-handed antineutrino. If v_L interacts with v_L^c then this results in a Majorana mass m_{LL}^v. Such mass terms appear in the Lagrangian density for the quantum field theory, where the bar over the v_L has a conventional meaning that need not concern us here. From our point of view here such mass terms may simply be regarded as interactions.

are electrically neutral. If such existed, however, they would violate the total lepton number L.

Although left-handed Majorana masses m_{LL} are possible in principle, in the Standard Model they are zero since the background Higgs field H^0 is incapable of flipping a v_L into a v_L^c. If the background Higgs field H^0 were a component of a Higgs triplet (H^{++}, H^+, H^0) instead of a Higgs doublet (H^+, H^0) then such a flipping would be possible. However, in the Standard Model only Higgs doublets are present and then H^0 can only flip a v_L into a v_R, as seen in figure 9. However, there is nothing to prevent the right-handed neutrinos v_R having Majorana masses M_{RR}, where the

Figure 12. If right-handed neutrinos v_R are added to the Standard Model, then they can also acquire a Majorana mass M_{RR} by coupling to their own charge and parity conjugated states v_R^c. Since it does not take part in the Standard Model weak interactions, its Majorana mass M_{RR} may be consistently taken to be much larger than the W, Z boson masses, and may be arbitrarily large. Such mass terms appear in the Lagrangian density for the quantum field theory, where the bar over the v_R has a conventional meaning that need not concern us here. From our point of view here such mass terms may simply be regarded as interactions that enable right-handed neutrinos to interact with left-handed antineutrinos.

magnitude of such masses can take any value, and in particular such masses could be very large. The Heisenberg Uncertainty Principle, which allows energy conservation to be violated on small time intervals, then allows a left-handed neutrino to convert into a heavy right-handed neutrino, via the Higgs interaction, for a brief moment before reverting back to being a left-handed neutrino, as shown in the lower diagram in figure 9. For a very large M_{RR}, this effectively results in a very small effective Majorana mass for the left-handed neutrino, $m_{LL}^{eff} = (m_{LR}^v)^2 / M_{RR}$. The presence of large right-handed Majorana masses M_{RR} therefore leads to an attractive mechanism for explaining the smallness of neutrino masses compared to charged fermion masses. This is the so-called see-saw mechanism. The smallness of the neutrino mass m_{LL}^{eff} is associated with the heaviness of the right-handed neutrino mass M_{RR}.

The third requirement for the absence of neutrino mass in the Standard Model is that the theory is renormalizable. This is a technical requirement that all the interactions of the theory are generated by particle exchange, and that quantum corrections to the theory do not introduce any infinities. A simple example of a non-renormalizable interaction that would generate neutrino mass would be a 'contact interaction' between two left-handed neutrinos and two Higgs fields, corresponding to the lower diagram in figure 9 but with the right-handed neutrino line shrunk to zero. In this case a Majorana mass $m_{LL}^{eff} = (m_{LR}^v)^2 / \Lambda$ would be generated but it would not be due to the exchange of heavy right-handed neutrinos but due to the non-renormalizable 'contact interaction' where the Standard Model is valid up to some cut-off Λ. In fact the lower diagram in figure 9, with very heavy right-handed neutrinos, is well approximated by such a non-renormalizable 'contact

interaction', since the M_{RR} is very large and its propagation length is very small, so in this case we would identify $\Lambda = M_{RR}$. Thus, even if non-renormalizable 'contact interactions' are added to the Standard Model they may be due to very heavy particle exchange. Of course the origin of the contact interaction may be due to the exchange of other particles of mass Λ, different from heavy right-handed neutrinos.

3.2 The see-saw mechanism

In this subsection we discuss the see-saw mechanism a little more quantitatively. Let us first summarize the different types of neutrino mass that are possible. There are Majorana masses of the form

$$m_{LL}\overline{\nu_L}\nu_L^c, \tag{15}$$

where ν_L is a left-handed neutrino field and ν_L^c is the CP conjugate of a left-handed neutrino field, in other words a right-handed antineutrino field. Such mass terms have been discussed in the previous section, and have been represented diagrammatically in figure 11. Strictly speaking such mass terms appear in the Lagrangian density for the quantum field theory, but from our point of view here they may simply be regarded as interactions that enable left-handed neutrinos to interact with right-handed antineutrinos, as depicted in figure 11.

Such Majorana masses are possible since both the neutrino and the antineutrino are electrically neutral and so Majorana masses are not forbidden by electric charge conservation. For this reason a Majorana mass for the electron would be strictly forbidden. However, such Majorana neutrino masses violate lepton number conservation, and in the standard model, assuming only the simplest Higgs bosons are present, are forbidden. The idea of the simplest version of the see-saw mechanism is to assume that such terms are zero to begin with, but are generated effectively, after right-handed neutrinos are introduced [38].

If we introduce right-handed neutrino fields then there are two sorts of additional neutrino mass terms that are possible. There are additional Majorana masses of the form

$$M_{RR}\overline{\nu_R}\nu_R^c, \tag{16}$$

where ν_R is a right-handed neutrino field and ν_R^c is the CP conjugate of a right-handed neutrino field, in other words a left-handed antineutrino field. In addition there are Dirac masses of the form

$$m_{LR}\overline{\nu_L}\nu_R. \tag{17}$$

Such Dirac mass terms conserve lepton number, and are not forbidden by electric charge conservation even for the charged leptons and quarks.

The Higgs mechanism, in its simplest form at least, forbids Majorana masses of the type m_{LL}^ν, involving the left-handed neutrino ν_L, and its CP conjugate ν_L^c, but permits Majorana masses M_{RR} involving purely right-handed neutrinos ν_R and its CP conjugate ν_R^c. In fact just as m_{LL}^ν must be zero in the Standard Model, so M_{RR} may be arbitarily large. The reason is essentially that the left-handed neutrino ν_L takes part in weak interactions with the W, Z bosons, and if it were very heavy it would disturb the theory. The right-handed neutrino ν_R, on the other hand, does not take part in weak interactions with the W, Z bosons, and so its mass M_{RR} can be arbitrarily large.

With the types of neutrino mass discussed in equations (16) and (17) (but not equation (15) since we assume no Higgs triplets) we have the see-saw mass matrix

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}. \tag{18}$$

Since the right-handed neutrinos are electroweak singlets the Majorana masses of the right-handed neutrinos M_{RR} may be orders of magnitude larger than the electroweak scale. In the approximation that $M_{RR} \gg m_{LR}$ the matrix in equation (18) may be diagonalized to yield effective Majorana masses of the type in equation (15),

$$m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T. \tag{19}$$

The effective left-handed Majorana masses m_{LL} are naturally suppressed by the heavy scale M_{RR}. In a one family example if we take $m_{LR} = M_W = 80$ GeV and $M_{RR} = M_{GUT} = 10^{16}$ GeV then we find $m_{LL} \sim 10^{-3}$ eV which looks good for solar neutrinos. Atmospheric neutrino masses would require a right-handed neutrino with a mass below the GUT scale.

With three families of left-handed neutrinos and three right-handed neutrinos the Dirac masses m_{LR} are a 3×3 (complex) matrix and the heavy Majorana masses M_{RR} form a separate 3×3 (complex symmetric) matrix. The light effective Majorana masses m_{LL} are also a 3×3 (complex symmetric) matrix and continue to be given from equation (19) which is now interpreted as a matrix product. From a model building perspective the fundamental parameters which must be input into the see-saw mechanism are the Dirac mass matrix m_{LR} and the heavy right-handed neutrino Majorana mass matrix M_{RR}. The light effective left-handed Majorana mass matrix m_{LL} arises as an output according to the see-saw formula in equation (19).

3.3 Sequential dominance

We wish to apply the see-saw mechanism to account for the atmospheric and solar mixing. A simple and natural way to achieve a neutrino mass hierarchy with large atmospheric

and solar mixing angles is the idea of sequential dominance [39]†.

3.3.1. Two state atmospheric mixing. It is instructive to begin by discussing a simple 2×2 example, describing the two state atmospheric mixing in subsection 2.1. The starting point is to assume that the right-handed Majorana mass matrix and the charged lepton mass matrix are diagonal‡, but the Dirac neutrino mass matrix is general, and then we write:

$$M_{\text{RR}} = \begin{pmatrix} Y & 0 \\ 0 & X \end{pmatrix}, \qquad m_{\text{LR}} = \begin{pmatrix} e & b \\ f & c \end{pmatrix}, \qquad (20)$$

where the Dirac mass matrix elements m_{LR} couple a particular Standard Model left-handed neutrino states ($v_{\mu L}$, $v_{\tau L}$), to a particular right-handed neutrino states (v_R^Y, v_R^X) labelled by its Majorana masses Y, X, respectively. For example the element b of m_{LR} corresponds to a Dirac mass coupling $v_{\mu L}$ to v_R^X. The see-saw formula in equation (19) $m_{\text{LL}} = m_{\text{LR}} M_{\text{RR}}^{-1} m_{\text{LR}}^{\text{T}}$ gives the effective left-handed Majorana mass matrix:

$$m_{\text{LL}} = \begin{pmatrix} \frac{e^2}{Y} + \frac{b^2}{X} & \frac{ef}{Y} + \frac{bc}{X} \\ \frac{ef}{Y} + \frac{bc}{X} & \frac{f^2}{Y} + \frac{c^2}{X} \end{pmatrix} \approx \begin{pmatrix} \frac{e^2}{Y} & \frac{ef}{Y} \\ \frac{ef}{Y} & \frac{f^2}{Y} \end{pmatrix}, \qquad (21)$$

where the approximation in equation (21) assumes that the right-handed neutrino of mass Y is sufficiently light that it dominates in the see-saw mechanism [41]:

$$\frac{e^2, f^2, ef}{Y} \gg \frac{b^2, c^2, bc}{X}. \qquad (22)$$

The left-handed Majorana mass matrix in equation (21) is now interpreted as a matrix of Majorana masses involving the Standard Model left-handed neutrino states ($v_{\mu L}$, $v_{\tau L}$) coupling to themselves. The physical neutrino masses m_i corresponding to the mass eigenstates v_i are obtained by diagonalizing the mass matrix in equation (21),

$$\begin{pmatrix} c_{23} & -s_{23} \\ s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \frac{e^2}{Y} & \frac{ef}{Y} \\ \frac{ef}{Y} & \frac{f^2}{Y} \end{pmatrix} \begin{pmatrix} c_{23} & s_{23} \\ -s_{23} & c_{23} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{e^2+f^2}{Y} \end{pmatrix}. \quad (23)$$

The neutrino mass spectrum from equation (23) then consists of one neutrino with mass $m_3 \approx (e^2 + f^2)/Y$ and one

---

†Note that sequential dominance also works if neutrinos are quasi-degenerate, providing one invokes an extended type II see-saw mechanism in which the zero in the see-saw matrix in equation (18) is replaced by a universal mass proportional to the unit matrix [39].

‡This is convenient since it means that one can identify the neutrino mixing angles with the mixing angles in equation (7) (recall that the lepton mixing matrix U in equation (6) involves the neutrino flavour eigenstates which share the same electroweak doublet as the charged lepton mass eigenstates, as discussed). However, neither of these assumptions is essential [40].

naturally light neutrino $m_2 \ll m_3$, since the determinant of equation (21) is clearly approximately vanishing, due to the dominance assumption. The atmospheric angle from equation (23) is $\tan \theta_{23} \approx e/f$ which can be large or maximal providing $e \approx f$. Thus, two crucial features, namely a neutrino mass hierarchy $m_3^2 \gg m_2^2$ and a large neutrino mixing angle $\tan \theta_{23} \approx 1$, can arise naturally from the see-saw mechanism assuming the dominance of a single right-handed neutrino [41].

3.3.2. Three family neutrino mixing. In order to account for the solar mixing angle, we must generalize the above discussion to the 3×3 case. The generalization of equation (20) is:

$$M_{\text{RR}} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix}, \quad m_{\text{LR}} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}. \quad (24)$$

The generalization of equation (22) is called sequential dominance [42]:

$$\begin{aligned} \frac{e^2, f^2, ef}{Y} &\gg \frac{a^2, b^2, c^2, ab, ac, bc}{X} \\ &\gg \frac{a'^2, b'^2, c'^2, a'b', a'c', b'c'}{X}, \end{aligned} \qquad (25)$$

where we also assume $d \ll e, f$. Ignoring the small primed terms, the see-saw formula equation (19) now gives a Majorana mass matrix in the basis (v_{eL}, $v_{\mu L}$, $v_{\tau L}$) as:

$$m_{\text{LL}} \approx \begin{pmatrix} \frac{a^2}{X} + \frac{d^2}{Y} & \frac{ab}{X} + \frac{de}{Y} & \frac{ac}{X} + \frac{df}{Y} \\ \frac{ab}{X} + \frac{de}{Y} & \frac{b^2}{X} + \frac{e^2}{Y} & \frac{bc}{X} + \frac{ef}{Y} \\ \frac{ac}{X} + \frac{df}{Y} & \frac{bc}{X} + \frac{ef}{Y} & \frac{c^2}{X} + \frac{f^2}{Y} \end{pmatrix}. \qquad (26)$$

Note that the lower 2×2 block of equation (26) can be identified with equation (21), so we expect to have large atmospheric mixing as before. The physical neutrino masses are obtained by diagonalizing the mass matrix in equation (26), which, ignoring phases, corresponds to performing the Euler rotation in figure 4 to go from the basis (v_e, v_μ, v_τ) to the basis (v_1, v_2, v_3), given by a generalization of equation (23):

$$U^{\text{T}} m_{\text{LL}} U = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \qquad (27)$$

where, from equation (7), $U = R_{23} R_{13} R_{12}$. After some algebra, in a small angle θ_{13} approximation, one finds [42] a full neutrino mass hierarchy

$$m_3 \gg m_2 \gg m_1 \qquad (28)$$

with

$$m_3 \approx \frac{e^2 + f^2}{Y}, \quad m_2 \approx \frac{a^2}{Xs_{12}^2}. \tag{29}$$

Note that sequential dominance can only account for a normal neutrino mass hierarchy, and not the inverted mass pattern. Assuming that d is negligible, the angles are determined to be [42]:

$$\tan\theta_{23} \approx \frac{e}{f}, \quad \tan\theta_{12} \approx \frac{a}{bc_{23} - cs_{23}},$$
$$\tan\theta_{13} \approx \frac{a(bs_{23} + cc_{23})}{m_3 X}. \tag{30}$$

Note that the solar mass and solar angle only depends on the sub-dominant couplings. In general large solar mixing can result. In particular tri-bimaximal mixing in equation (12) results from the choice $e = f$, $a = b = -c$ which is called constrained sequential dominance [43].

3.4 Large extra dimensions

An alternative explanation of small neutrino masses comes from the concept of extra dimensions beyond the three that we know of, motivated by theoretical attempts to extend the Standard Model to include gravity [44]. According to string theory, there may be six extra space dimensions in addition to the three space and one time dimension of the Standard Model. Indeed it has been suggested that the Standard Model lives on a 3 space dimensional brane, and that we are analogous to a bug walking on the surface of a pond, supported by the membrane of the water, as shown in figure 13. The extra dimensions are 'compactified' (rolled up) on circles of small radius R so that they are not normally observable. Such extra dimensions if uniformly compactified are called 'flat' or if the compactification involves a distortion or warping are called 'warped'. It has been suggested that right-handed neutrinos (but not the rest of the Standard Model particles) experience one or more of these extra dimensions. The right-handed neutrinos then only spend part of their time in our world, leading to very small Dirac neutrino masses [45]. In such theories there is a relation between the usual four-dimensional Planck mass $M_{\mathrm{planck}} \sim 10^{19}$ GeV/c^2, the string scale M_{string} and the compactification radius of the 'flat' extra dimensions R given by:

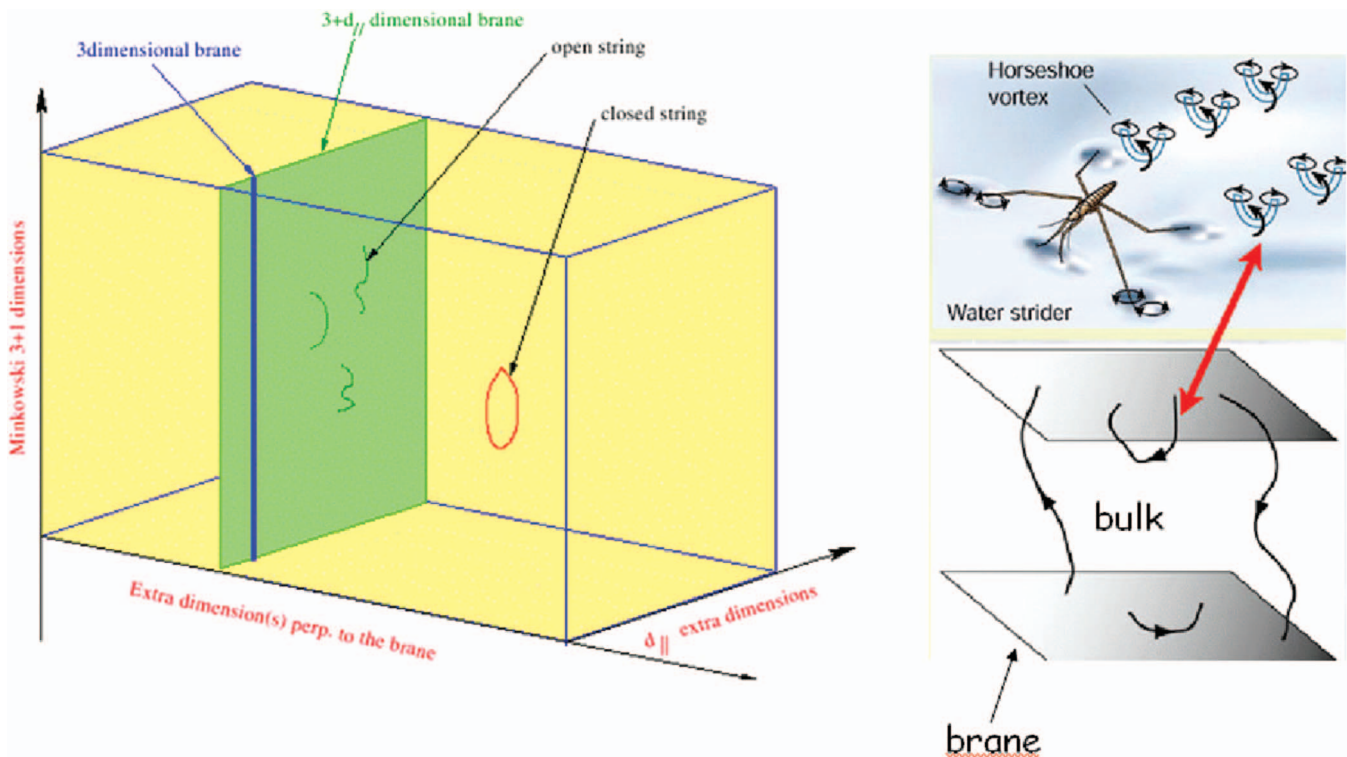$$M_{\mathrm{Planck}}^2 = M_{\mathrm{string}}^{2+n} R^n, \tag{31}$$

Figure 13. According to string theory there may be extra dimensions in addition to the $3+1$ of the Standard Model. According to the latest theories, the Standard Model may reside on a 3 space dimensional brane. Open strings attach themselves to such branes, while closed strings float freely in the bulk of all the extra dimensions, as indicated in the left-hand panel taken from [44]. Thus, we may appear as this water strider in the right-handed panel (taken from a talk by Jean Orloff) walking over the membrane of the water on a pond, unaware of the bulk of water below us. Reprinted figure 1 from *High Energy Physics* (2005), with permission from I. Antoniadis.

where there are n extra dimensions. For example, for one extra dimension the right-handed neutrino wavefunction spreads out over the extra dimension R, leading to a suppressed Higgs interaction with the left-handed neutrino, with a suppression factor of $1/(M_{\text{string}}R)^{1/2}$. This corresponds to the coupling between left- and right-handed neutrinos being more suppressed for larger R, as the right-handed neutrino spends less of its time on the 3 space dimensional brane where the left-handed neutrino lives the larger R becomes. The Dirac neutrino mass is therefore suppressed relative to the electron mass, and may be estimated as:

$$m_{\text{LR}}^{v} \sim \frac{m_e}{(M_{\text{string}}R)^{1/2}} \sim \frac{M_{\text{string}}}{M_{\text{Planck}}} m_e, \qquad (32)$$

where we have used equation (31). Clearly low string scales, below the Planck scale, can lead to suppressed Dirac neutrino masses. Similar suppressions can be achieved with anisotropic compactifications [46].

3.5 *Cosmology*

What neutrinos lack in mass they make up for in number. The Universe is filled with neutrinos, created less than a second after the Big Bang, with each cubic centimetre of space containing about 112 neutrinos of each species, giving more than 300 in total. This makes neutrinos the second most abundant particles in the Universe after the Cosmic Microwave Background (CMB) photons, where each cubic centimetre of space contains 411 photons. The CMB photons are the remnant of the Big Bang fireball, originating from a time some 380 000 years after the Big Bang when the Universe had cooled sufficiently to enable the first atoms to form, thus rendering the Universe transparent. By contrast there are on average a billion times fewer electrons or protons that survived the great annihilation of matter and antimatter which occurred about a second after the Big Bang. Indeed Cosmology today presents three major puzzles: why was there any excess of matter over antimatter in the Universe; what is the major matter constituent of the Universe; why is the Cosmological Constant extremely small? Massive neutrinos may hold important clues.

Matter and antimatter would have been created in equal amounts in the Big Bang but all we see is a small amount of excess matter, corresponding to about one electron or proton for every billion photons (or neutrinos) in the Universe. In order to generate matter–antimatter asymmetry in the Big Bang, Sakharov in 1967 proposed a set of three necessary conditions: baryon number violation; C and CP violation; and a violation of thermal equilibrium. Perhaps surprisingly, the Standard Model can satisfy all of these conditions, for example baryon number is violated by

non-perturbative effects called sphalerons, but detailed calculations show that it cannot lead to the desired matter–antimatter asymmetry [47]. The see-saw mechanism allows for a novel resolution to this puzzle. The idea, due to Masataka Fukugita and Tsutomu Yanagida of Tokyo University [48], is that when the Universe was very hot, just after the Big Bang, the heavy right-handed neutrinos would have been produced, and could have decayed preferentially into leptons rather than antileptons, a possibility that is allowed since right-handed neutrinos have Majorana masses that violate lepton number L, neutrino interactions also may violate CP. The excess leptons may subsequently be converted into an excess of baryons via the Standard Model sphaleron effects mentioned above. The see-saw mechanism therefore opens up the possibility of generating the baryon asymmetry of the Universe via 'leptogenesis'. This process clearly requires CP violation for neutrinos, and increases the motivation to discover leptonic CP violation experimentally. However, as discussed, this would require Superbeams or a Neutrino Factory.

Studies of the kinematics of galaxies and galaxy clusters suggest that at least 90% of the mass of the Universe is made of unknown dark matter. Cosmology is sensitive to the absolute values of neutrino masses, in the form of relic hot dark matter, where the dark matter is 'hot' in the sense that the neutrinos were relativistic at the epoch of galaxy formation. Such hot dark matter tends to lead to less clumpiness of galaxy clusters, due to the free streaming effects of such relativistic particles which tends to wash out galaxy structures. On the other hand, particles more massive than neutrinos, known generically as weakly interacting massive particles (WIMPs), can behave as cold dark matter (non-relativistic at the epoch of galaxy formation) which tends to increase the clumpiness of galaxy clusters. Such WIMPs can in principle be directly detected in underground laboratories [49].

Recent results from the Cosmic Microwave Background (CMB) experiments (especially WMAP [50]) and galaxy redshift surveys (especially 2dF and SDSS [51]), gives a strong preference for cold dark matter over hot dark matter. This leads to a limit on the amount of hot dark matter that can be accommodated. When combined with oscillation data, this leads to an upper limit on the absolute mass of each neutrino species of about 0.3 eV [52], corresponding to the sum of neutrino masses being less than about 1 eV, as shown in figure 14. More aggressive bounds are claimed when other data are taken into account, also as shown in figure 14. Neutrinos could constitute anything from 0.1% to 2% of the mass of the Universe, corresponding to the heaviest neutrino being in the mass range 0.05 to about 0.3 eV. Neutrinos any heavier than about 0.3 eV, corresponding to the sum of neutrino masses exceeding about 1 eV, would lead to galaxies being less clumped than actually observed by the recent galaxy
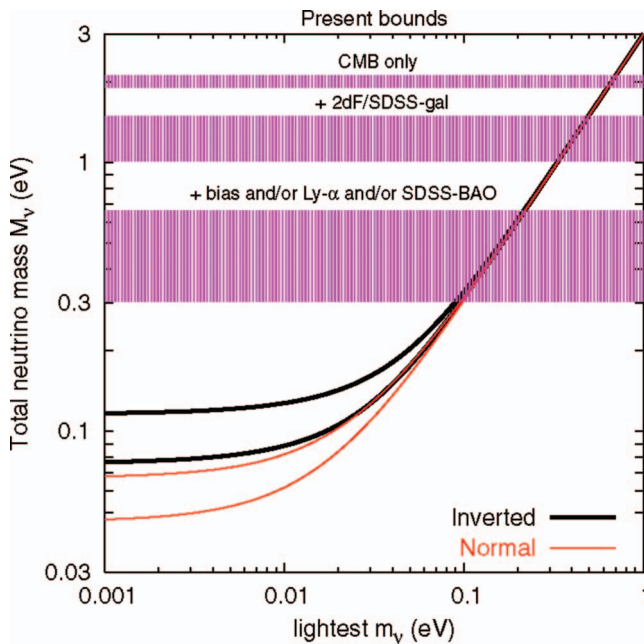
Figure 14. The current bound on the sum of the three neutrino masses at the 2σ level from cosmology using CMB only (upper band), CMB combined with galaxy survey data (centre band), and then including other effects (lower band) as discussed in [52] from where this figure is taken. Also shown are the expected ranges for the sum of the three neutrino masses as a function of the lightest neutrino mass from neutrino oscillation data for both a normal mass ordering (red lines) and an inverted mass ordering (black lines). Reprinted from *Phys. Rept.* **429**, J. Lesgourgues and S. Pastor, Massive neutrinos and cosmology, pp. 307–379. Copyright 2006, with permission from Elsevier.

redshift surveys. Ambitious claims are made that future CMB measurements from the Planck satellite, due to be launched in 2008, including the effect of weak gravitational lensing (the deduction of large scale matter distributions from CMB anisotropy) could constrain the sum of neutrino masses down to 0.05 eV, corresponding to the atmospheric neutrino mass in hierarchical models. This illustrates the breathtaking rate at which neutrino physics continues to advance.

4. Conclusion

Since the discovery of neutrinos, just over half a century ago, we have learned much about neutrinos, yet to a large extent neutrinos remain somewhat elusive, if abundant, members of the Standard Model families of quarks and leptons. Although we are not made of neutrinos, the Universe as a whole is, with over a billion neutrinos for every single atom. Stars, such as the Sun, would not burn without neutrinos, nor would supernovae explode, producing the star dust from which we are made.

Almost a decade ago there was a revolution in neutrino physics when they were found to have a tiny, but non-zero, mass, in contradiction with the Standard Model. It is fair to say that the past decade has been a golden age of neutrino physics, with huge progress in neutrino physics both on the experimental and theoretical fronts. The surprise discovery of neutrino mass in atmospheric neutrinos, was also accompanied by the further surprise of large, possibly maximal, neutrino mixing. The solar neutrino 'problem' is no more, instead we have the discovery of solar neutrino 'mass', again involving large mixing.

These large mixings, large compared to the corresponding quark mixing angles, can be understood from the see-saw mechanism, for example by the sequential dominance mechanism, but the see-saw mechanism is very difficult to test. Nevertheless, if the see-saw mechanism in its simplest form is correct, then it could explain the matter–antimatter asymmetry in the Universe via the leptogenesis mechanism. However, this would require both Majorana masses and CP violation, neither of which have been experimentally established. Alternatively, if neutrinos have Dirac masses, then their smallness might be accounted for by invoking large extra dimensions.

Although neutrino physics has now entered the age of precision measurements, much is left to learn. For example the CP violating phase δ is yet to be measured, and the reactor angle θ_{13} is similarly undetermined. The neutrino mass ordering is not yet specified either, nor is the absolute scale of neutrino mass, and even the nature of neutrino mass itself has not been verified. The answer to all these questions will have profound implications for particle physics and cosmology.

Given the recent results from MiniBooNE, which failed to confirm the LSND signal, one might be tempted to think that the golden age of major surprise discoveries in neutrino physics is over. However, it is just possible that neutrino physics has further surprises up her sleeve. For example, neutrinos may yet be observed to have rather large Majorana masses, saturating or even violating the cosmological bounds. The recent controversial claim of a signal in neutrinoless double beta decay is being vigorously checked, and if confirmed would imply that neutrino masses have quite a high degree of degeneracy. This would certainly set the cat amongst the cosmological and theoretical pigeons, and herald a new neutrino revolution to rival the one described here.

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