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Supersolidity and superfluidity

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Supersolidity and superfluidity

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Is it possible that a solid flows like a liquid, or even without friction like a superfluid? At first sight, the crystalline order in real space looks contradictory with the coherent motion which is responsible for superfluidity. However, several authors considered this possibility in 1969 – 1970, and experiments done by Kim and Chan in 2004 suggest that solid ^{4}He is indeed a superfluid solid—a 'supersolid'—below about 100 mK. This article summarizes my present understanding of this paradoxical issue.

1. Why looking for mass flow through a solid?

Let us start with a little reminder of Italian science in the 17th century. Evangelista Torricelli was a friend of Galileo. He is famous for having done the first experimental measurement of the atmospheric pressure in 1643 – 1644. For this he filled a glass tube with liquid mercury and immersed it in a mercury bath (see figure 1). With vacuum at the closed top of the tube—in reality a very small vapour pressure—he observed an equilibrium between the atmospheric pressure outside and the weight of a liquid mercury column 760 mm high. This is because the liquid was able to flow, consequently to reach a so-called 'hydrostatic equilibrium' along its vertical column. Now, when he evacuated the outside with a pump (see the right part of figure 1), the level was the same in the two communicating tubes, because the temperature was the same on both sides, consequently the saturated vapour pressure as well. What allows the equilibration of levels is of course the ability of liquid mercury to flow. In the spring of 2006, we ran a similar experiment with solid helium at 50 mK [1] instead of liquid mercury at 300 K. I wish to explain why we tried such a surprising thing.

By definition, a solid resists the application of a shear stress. Imagine a solid between two horizontal plates and push the top plate to the right while pushing the bottom one to the left. The solid deforms elastically and if you release the shear stress it comes back to its original shape.

In the same circumstances, a fluid would flow and never flow back when the stress is released. As a consequence, it is natural to find it to be contradictory that a solid could flow as a liquid. But, in fact, we know that the solid or liquid character of a piece of matter depends on time, also on the strength of the applied force. If you blow gently on soap foam, it reacts as an elastic medium. But, of course, if you stir it with a spoon, it undergoes plastic deformation. A foam is an elastic solid at low stress and a viscous liquid at high stress. Similarly, a block of iron is obviously a solid in usual conditions but iron wires are made by extrusion, that is by pushing the metal through a hole without melting it. Even at temperatures below the melting point of iron, but under sufficiently high stress, iron behaves like a viscous liquid. We could also think to the ice flowing down in glaciers, or even to the Earth's crust forming mountains. Many materials are more or less 'visco-elastic' as one says.

The surprising thing with flowing a solid under the application of just some hydrostatic pressure difference is thus that this is a very small stress: usually solids do not flow under their own weight, they keep their original shape and they do not spread to form any kind of flat pancake or film. But the question we are asking is even more striking. Could a solid flow *without any friction*, like a superfluid? For reasons due to its quantum character, *liquid* helium can flow without dissipation. That is why it was called 'superfluid' by Piotr Kapitza, one of the scientists who discovered it in 1937 [2,3]. As a consequence, superfluid

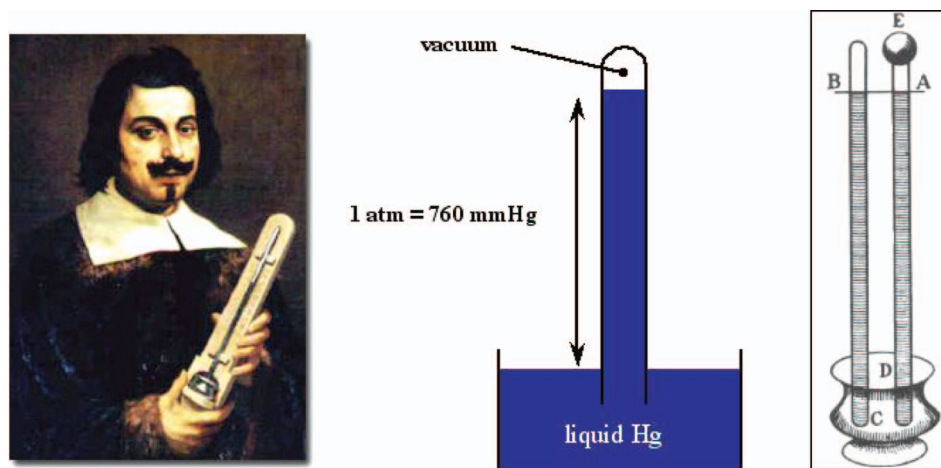*Corresponding author. Email: balibar@lps.ens.fr

Figure 1. In 1643–1644, Evangelista Torricelli (left) measured the atmospheric pressure for the first time. For this he immersed a glass tube in a mercury bath and observed an equilibrium between the atmospheric pressure outside and the weight of a mercury column 760 mm high (centre image). When he evacuated the outside with a pump, he observed that the ability of the liquid to flow forced the two levels to be the same (right). In 2006, Sasaki *et al.* [1] ran a similar experiment with solid helium in order to test its supersolid properties.

helium can flow at a non-zero velocity even if no force is applied to it. This behaviour is highly non-classical. It happens only below a certain threshold called its 'critical velocity' and, if one applies a force, for example a hydrostatic pressure difference across a fine capillary, a superfluid flows at a so-called 'critical velocity' which is constant, independent of the pressure difference. Furthermore, the thinner the capillary through which superfluid flow is observed, the faster it flows. In channels only a few Angströms thick, where normal fluids would be completely blocked by their viscosity, superfluid helium flows at meters per second! This was progressively understood as a consequence of the quantum properties of superfluids. Their atoms or molecules are 'condensed' in the sense of Bose and Einstein [4,5] so that they lose their individual character and form a macroscopic wave of matter which moves as a whole. Bose–Einstein condensation being also understood as the appearance of order in momentum space, one could say that all atoms move at the same velocity but that their individual positions are unknown, delocalized in the whole sample. From this point of view it is doubly paradoxical to imagine that a solid is superfluid: not only would it flow without dissipation under the application of a *small* force, it would also be ordered both in ordinary space (the atoms would be localized on some sites) and in momentum space (the atoms would be delocalized waves moving in a coherent way). The latter looks like a violation of Heisenberg's uncertainty principle.

2. The experiment by Kim and Chan

In 2004, Eunsong Kim and Moses Chan performed a very surprising experiment [6,7]. They made a cylindrical box,

10 mm in diameter, which was suspended by a thin tube and could oscillate around it. This is a so-called 'torsional oscillator' whose period of oscillation could be measured with great accuracy. The box could be filled with helium through the tube. The period depends on the total momentum of inertia, consequently on the helium mass which is driven by the box walls. Kim and Chan filled their box with liquid helium at high pressure and high temperature and then cooled it down. At a few degrees, the liquid solidified in the box and they cooled further down. To their great surprise, they found that, below a certain critical temperature—typically 100 mK—the oscillation period suddenly decreased as if 1% of the helium mass decoupled from the oscillating motion of the walls. A superfluid would do that because its ground state is immobile, and it would do it below a critical velocity of the walls. Kim and Chan found that their phenomenon— they called it 'supersolidity'—occurred only if the walls moved not faster than 10 μm s^{-1}. ^{4}He atoms being Bose particles, liquid ^{4}He can be superfluid. Conversely, ^{3}He atoms being Fermi particles, liquid ^{3}He needs its atoms to form pairs for superfluidity to appear, something which happens only at very low temperature [8]. Kim and Chan repeated their experiment with solid ^{3}He in the same box and found no decoupling. Then they put a little additional wall inside the box in order to interrupt the circulation of mass around the rotation axis. They found again that the decoupling vanished. In the rotating frame of the box walls, one percent of the helium mass was really rotating through the rest of the solid as a superfluid! Facing such a series of observations, Kim and Chan proposed an interpretation based on a series of papers published 35 years ago by Andreev and Lifshitz, Chester and Leggett.

3. Early theories of supersolidity

In 1969, Andreev and Lifshitz [9] proposed the following scenario. Suppose that there are empty sites—so called 'vacancies'—in helium crystals. A classical vacancy is a hole which does not move much. It has a certain energy E_0 which depends on the interatomic potential. But helium crystals are quantum in the sense that the quantum fluctuations of atoms on their sites is unusually large. A vacancy has thus a non-zero probability to exchange its position with a neighbouring atom by quantum tunnelling. As a result vacancies are not localized objects but delocalized waves in the crystal where they form an energy band, from $(E_0 - \Delta)$ to $(E_0 + \Delta)$ like electrons in a metal. If the tunnelling frequency from site to site is large, the width 2Δ of the energy band is also large. Now suppose that Δ is larger than E_0, the energy of a localized vacancy. Then the bottom of the energy band is below zero so that, even at zero temperature, there should be a finite density of vacancies in helium crystals. This is an astonishing situation: the crystal is a mass density wave which is periodic in three-dimensional space but the ground state is such that the number of atoms is different from the number of sites. Some authors call this an 'incommensurate' crystal. Now, if there is a non-zero density of vacancies in ^{4}He crystals at arbitrarily low temperatures, these vacancies should form a Bose–Einstein condensate and become superfluid below some critical temperature T_c. Finally, since a vacancy moving in one direction is equivalent to an atom moving in the opposite direction, ^{4}He crystals could realize the surprising coexistence of a crystalline order in real space and superfluid mass flow through the crystal lattice due to the ordering of vacancies in momentum space.

According to Andreev and Lifshitz, Penrose and Onsager had thus incorrectly concluded in their famous 1956 article [10] that superfluidity was impossible in a crystal. Bose–Einstein condensation (BEC) had been introduced for ideal gases with no interactions, and the 1956 article is a development of Onsager's generalization of BEC to dense interacting systems like liquid helium where interactions are strong. They describe the long-range quantum coherence which characterizes a superfluid in a particular formalism called the 'one-body density matrix'. Long-range correlations mean that atoms a long distance away from each other do not move independently but as waves in phase. Correlations show up in the behaviour of a particular off-diagonal term of the density matrix, and that is why BEC was generalized as 'off-diagonal long-range order' (ODLRO). Near the end of their article, Penrose and Onsager had claimed that ODLRO was impossible in crystals and everybody believed them.

However, Chester [11] noticed that the argument by Penrose and Onsager was based on the use of particular wave functions which were not symmetrized, although they should for bosons. By using other wave functions ('Jastrow' wave functions), he explained that ODLRO was in fact possible in crystals, something which had already been noticed by Reatto [12]. Shortly afterwards, Tony Leggett [13] improved on Chester's work and predicted that a crystal with ODLRO should rotate in an anomalous way like ordinary superfluids. These strange rotation properties are called 'non-classical rotational inertia' (NCRI). What happens when one tries to rotate a superfluid in a bucket? At first, that is if the rotation speed is small enough, the superfluid is not driven by the bucket walls. It stays at rest. This is because a superfluid cannot rotate at an arbitrary velocity. Vortices are quantized. The circulation of the velocity is necessarily equal to an integer multiple of a particular quantum which is the ratio (h/m) of Planck's constant to the mass of He atoms. When rotating the bucket faster, quantum vortices nucleate (usually on the walls) and enter the fluid. There is some dissipation of energy associated with the presence of vortices but below the critical velocity at which vortices appear, the fluid motion is dissipationless. When Kim and Chan observed the strange properties of their solid ^{4}He sample, they proposed that it was related to Leggett's NCRI.

4. Later and recent theoretical developments

In 1970–1975, Matsuda and Tsuneto [14] followed by Imry and Schwartz [15] raised an important question: Is supersolidity possible in the absence of vacancies or does it require that the solid is 'incommensurate'? I am not quite sure that this issue is solved. When a liquid crystallizes, there is a spontaneous symmetry breaking: the liquid is invariant by translation while the crystal is periodic in space. But it is not obvious that the atoms are individually trapped in the periodic potential of the others. Could there be a kind of collective motion of the atoms by quantum tunnelling through their own lattice? Chester ruled this out for Jastrow wave functions so that, for BEC to exist in a crystal, it looked necessary to have vacancies. Imry and Schwartz extended Chester's proof to a wider class of wave functions so that it looked more firmly established that a crystal without free vacancies could not be a supersolid. At this stage I write *free* vacancies because there could be vacancy-interstitial pairs which are the result of one atom being slightly displaced from its equilibrium position. Note however that not everybody agrees that free vacancies are required for supersolidity [16].

When the two papers by Kim and Chan appeared, the same question was considered again. Prokof'ev and Svistunov [17] claimed that it is a theorem: supersolidity is impossible in the absence of free vacancies or interstitials or both in the limit of zero temperature. However, a numerical simulation by Galli and Reatto found a non-zero

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superfluid fraction in a commensurate helium crystal [18]. They used particular wave functions named 'shadow wave functions' for this and explained that, since the properties of real helium crystals were very well reproduced by using these wave functions, one should be confident that supersolidity exists in the absence of free vacancies. A few months later, Bryan Clark and David Ceperley [19] published another simulation, this time performed with a 'path integral Monte Carlo method' (PIMC). They explained that the calculation of ODLRO by variational methods is biased by the choice of trial wave functions while their method is not. They found that the ground state of helium crystals is commensurate and shows no ODLRO, that is no supersolidity. Both Prokof'ev and Svistunov on the one hand, and Clark and Ceperley on the other hand concluded that if supersolidity exists, it has to be somehow related to defects in helium crystals, it cannot be a fundamental property of perfect crystals.

From a theoretical point of view, one would have considered the situation as clear if a very interesting paper had not been published by Anderson *et al.* [20]. This paper reanalyses old measurements by Fraass *et al.* [21] of the temperature variation of the lattice spacing a in ^{4}He crystals. At that time, they had interpreted the variation as a consequence of the thermal activation of vacancies. A fit with an Arrhenius exponential of the observed increase of $a(T)$ had provided an activation energy for the vacancies of order 10 K. This result is consistent with numerical estimates and, if true, it means that, at a temperature of 0.1 K, the probability that a vacancy occupies a given lattice site is proportional to $\exp -100$, which is totally negligible. However, at 1 K, vacancies should exist and manifest themselves as an exponential term in the specific heat. But no such contribution has ever been measured. Instead, a surprising T^7 term was found as the first correction of the usual T^3 contribution from phonons. This had been a long standing puzzle. Anderson *et al.* explained that such a T^7 term is precisely what is expected if there is a finite density of vacancies at zero temperature, in which case the lattice spacing should increase as T^4, a simple power law which fitted experimental data as well as Simmons' exponential. However, Anderson *et al.* explained that the density of vacancies in ^{4}He crystals is much too small for a Bose–Einstein condensation at 100 mK. Furthermore, a recent calculation by Maris and Balibar [40] shows that the T^7 term is more likely due to the dispersion of phonons.

Many other articles have been published on this confusing issue. For example Boninsegni *et al.* [22] showed that a non-crystalline solid—a ^{4}He glass—should be supersolid. de Gennes proposed that the elastic properties of ^{4}He crystals could be anomalous, due to dislocations being highly mobile at low temperature [23]. Given all these contradictory articles, the least one could say is that the

theoretical understanding of supersolidity is far from clear. This is probably what makes it so interesting. As we shall now see, experimental results are also somewhat confusing.

5. Experiments

Torsional oscillators have been extensively used to study superfluidity, especially in Reppy's group at Cornell University. Together with Anne Sophie Rittner [24] in 2006, John Reppy repeated the experiment by Kim and Chan. They looked for possible effects of disorder and observed a very important phenomenon. They found that supersolid behaviour disappeared if the crystals were annealed. They first grew their crystals by the same 'blocked capillary method' as Kim and Chan and observed a similar change in oscillating period at low temperature. After warming these crystals up to 1.5 K (this is hot for helium crystals!) for 13 h, the effect disappeared. When growing the crystal fast again, the effect reappeared. They concluded that supersolidity was necessarily related to the presence of defects in ^{4}He crystals. However, Kim and Chan [25] also tried annealing their crystals and found that the supersolid behaviour was enhanced! As for Kondo *et al.* [26], they found a supersolid fraction about 3 times smaller than found by Kim and Chan and no effect of annealing. What shall we conclude from these contradictory results? In fact, there is a definite scatter in Kim and Chan's data, even though they explain [25] that, thanks to a better crystal preparation method, this scatter has been considerably reduced. In my opinion, the simple fact that various experimental results are not quantitatively reproducible indicates that supersolidity depends on the history and preparation of crystals, consequently that it is related to quenched disorder. However, there is not yet a universal consensus on this statement.

In my opinion, the study by Kim and Chan of the effect of ^{3}He impurities is also consistent with the importance of defects in the whole issue. They performed a series of measurements as a function of impurity concentration [7]. They found that, even in the small concentration range from 10^{-7} to 10^{-4}, ^{3}He impurities had a large effect on both the supersolid fraction and the onset temperature T_c for mass decoupling. Day and Beamish [27] noticed that this was another indication that supersolidity is related to the existence of defects (dislocations, grain boundaries) where ^{3}He impurities adsorb so that their local concentration is much higher than the average bulk concentration.

Two other observations contradict the simple model of supersolidity in terms of Bose–Einstein condensation of vacancies. If BEC took place in the crystal bulk, the specific heat should show a large singularity similar to the famous 'lambda' singularity observed in bulk superfluid helium. Kim and Chan found a very small anomaly in the specific heat of solid helium but nothing like the expected lambda-

singularity [28]. A very precise measurement of the melting curve was also done by Todoshchenko *et al.* [29] who found no contribution to the solid entropy other than the usual one due to phonons.

Finally, Kim and Chan observed a rather puzzling variation of the supersolid density with pressure [25]. The energy of a localized vacancy should increase strongly with pressure. The tunnelling frequency of quantum vacancies should decrease because the energy barrier separating neighbouring sites should increase. As a result, one expects the bottom of the energy band of vacancies to increase with pressure and their density to decrease. Conversely, Kim and Chan have observed that their supersolid density had a non-monotonic variation with pressure: it increased from 26 to 55 bar before decreasing above.

At this stage, we tried to look for mass flow through ^{4}He crystals in a simpler experiment 'a la Torricelli', which would provide more direct evidence of supersolid behaviour. In fact, at least two groups had already attempted to observe a dc-mass flow through solid helium. Day and Beamish filled a piece of porous glass with solid ^{4}He and increased the pressure outside in order to see if more mass entered [30]. At low temperature, they observed no additional mass flowing in. However, one could argue that, in their setup, mass flow required some deformation of the crystal lattice. It appeared interesting to test the possibility of mass flow through a fixed lattice. In 1989, Bonfait *et al.* [31] tried to fill a thin cylindrical capacitor with solid ^{4}He at the liquid–solid equilibrium pressure P_m and at low temperature ($T < 20$ mK) (see figure 2). For the solid to grow inside, mass had to flow out because the liquid density is smaller by ten percent and the top of the capacitor was closed. This time, no deformation of the crystal lattice was necessary. But the solid never grew inside Bonfait *et al.*'s capacitor. Although this strongly supported the absence of dc-mass flow through their crystals, Bonfait *et al.*'s negative result could have been due to facets blocking the growth at the entrance of the capacitor. This is because crystals are fully facetted during growth and facets easily anchor on wall defects [32]. We thus designed a similar experiment were mass flow would be due to the melting of a crystal [1], in which case it is known that facets disappear and the liquid–solid interface is much less pinned to walls [32].

6. The experiment by Sasaki *et al.*

As shown in figures 2 and 3, we have immersed a glass tube in solid ^{4}He at P_m, the solid–liquid equilibrium pressure of ^{4}He. In such experiments, the temperature is sufficiently homogeneous in the cell for the crystal shape to be determined by gravity and surface tension only. As a result, the crystal occupies the lower part of the cell as would do water in a glass. Crystals were grown from the superfluid liquid in the range 1.3 to 1.4 K by adding mass through a fill line connecting the cell to a high pressure cylinder outside. By growing them fast enough we could solidify part of the inside of the glass tube (see [1] for more details). After this, the whole cell was cooled down to 50 mK to look for mass superflow. For this we quickly melted the crystal outside the tube by pumping some helium through the fill line. This way, we could obtain a situation with the level of the liquid–solid interface higher by 1 cm inside the tube than outside. To reach the hydrostatic equilibrium where the two levels are at the same height, the solid inside had to melt and mass to flow outside since the liquid density is 10% lower that that of the solid. Note that any hydrostatic pressure difference is so small (less than 2×10^{-4} bar) that no plastic deformation of the solid could occur. But we expected the two levels to equilibrate if supersolidity took place.

In this experiment, we studied 13 crystals with various qualities. The fast growth at 1.3 K usually creates grain boundaries. Each grain boundary makes a groove, a kind of inverted cusp where it meets the solid–liquid interface (see figures 3 and 4). This is a consequence of mechanical equilibrium because a grain boundary has a non-zero
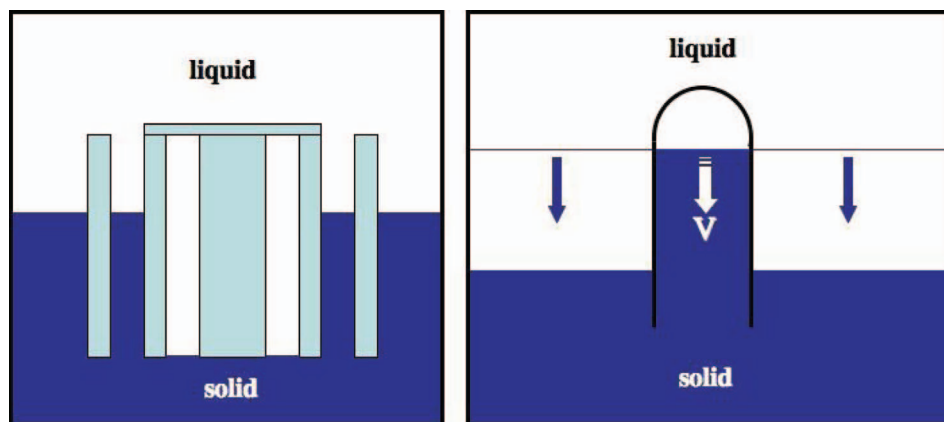

Figure 2. Left: a schematic representation of the experiment by Bonfait *et al.* Right: the glass tube used by Sasaki *et al.* [1].
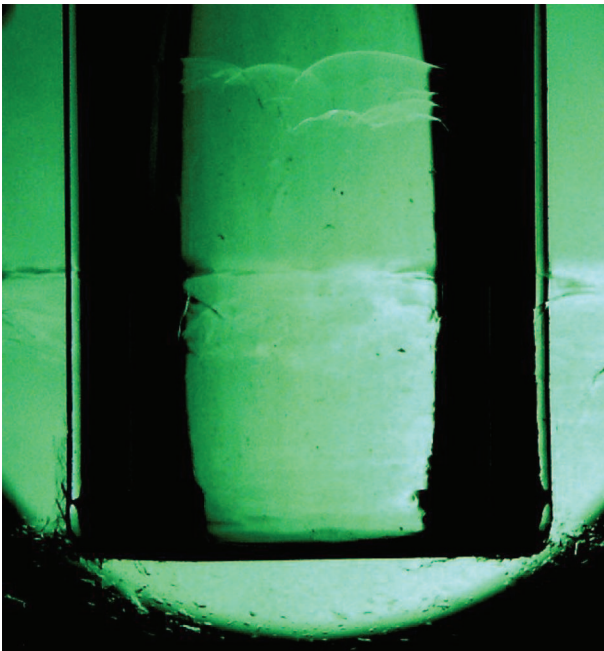
Figure 3. Photograph of solid helium prepared at 1.3 K by Sasaki *et al.* [1]. The interface between the liquid and the solid shows cusps where grain boundaries emerge. Outside the tube which appears black because of refraction effects, the interface is lower than inside. It goes all around the tube so that it can be seen on the right and on the left of the tube, but also in front of it.
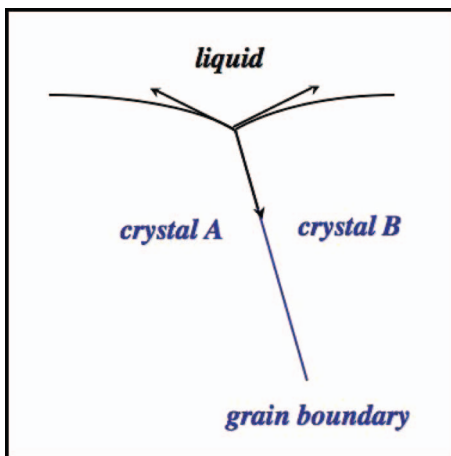

Figure 4. A grain boundary makes a groove, a kind of inverted cusp where it emerges at the solid–liquid interface. This is because it has a certain surface tension which needs to be equilibrated by the liquid–solid interfacial tension for mechanical equilibrium.

surface tension like the solid–liquid interface. Although we could not see the grain boundaries inside the crystal, we could know of their existence from the presence of these

cusps. Just after growth we could see many cusps but most of them disappeared in a time in the order of 1 hour. It seems that grain boundaries are pinned to defects at the glass surface. There must be some stress gradients in these crystals which slowly evolve in time so that, suddenly, some grain boundaries unpin and quickly move to the side walls where they vanish. Sometimes a few cusps remained for a long time so that we could study crystals with some cusps, some others with no cusps.

For good quality crystals showing no cusps inside the tube we observed no mass flow. The solid–liquid interface inside the tube remained fixed within 50 μm over 4 h. This sets an upper bound for mass flow through the crystals: it is at least 300 times less than expected if these crystals had a 1% concentration of vacancies moving at a critical velocity of 10 μm s^{-1}, as first proposed by Kim and Chan. Our observation shows that, if supersolidity exists in the bulk of ^{4}He crystals, it cannot result from such a simple mechanism. Furthermore, it also shows that, if there is a mass flow at the interface between the glass wall and these crystals, as proposed by Dash and Wettlaufer [33], it also has to be very small.

With three crystals showing cusps inside the tube, we observed some mass flow. Crystal 1 had one cusp and relaxed by 1 mm until the cusp disappeared and the relaxation stopped. This shows that grain boundaries are necessary for mass flow. The grain boundaries have to somehow connect the liquid inside the tube to the liquid outside the tube. Crystal 2 had more cusps. It relaxed faster and down to the equilibrium where the levels are the same inside and outside the tube. As shown by figure 5, the interface moved first at 6 μm s^{-1}; then it reached an even more disordered region of the crystal after 500 s, where more cusps were visible, and the velocity changed to 11 μm s^{-1}. After about 20 min, the level height difference $h(t)$ reached 0 and stopped changing. Classical dissipative motion would lead to an exponential relaxation because the velocity would be proportional to the applied force (dh/dt would be proportional to h). Conversely, here the relaxation is linear. This is typical of superfluid flow at a critical velocity. This second observation is consistent with the work of Burovski *et al.* [34] who predicted that the matter inside grain boundaries is superfluid within a very general model of quantum solids. During the writing of this article, Pollet *et al.* confirmed this with Monte Carlo simulations of real helium crystals [35]. Clark and Ceperley found the same three atomic layers thickness for grain boundaries (see figure 6) but they have not yet published results on their superfluidity.

Coming back now to the first crystal which had only one cusp, we estimated the critical velocity along the grain boundary from the velocity of the relaxation (0.6 μm s^{-1}). There are some unknown parameters like the thickness e of a grain boundary compared to the interatomic spacing a, its
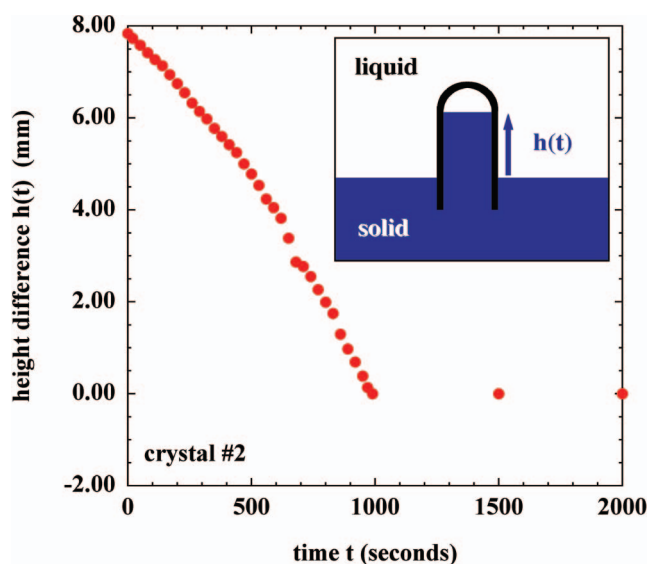
Figure 5. The relaxation of the level difference $h(t)$ for 'crystal 2' in the experiment by Sasaki *et al*. The level relaxes at 6 μm s^{-1} during the first 500 s and then at 11 μm s^{-1} until it reaches the equilibrium at $h = 0$ where it stops. A viscous fluid would relax exponentially. Here the relaxation is allowed by superfluid flow through grain boundaries which are more numerous in the second half of the relaxation.

width w compared to the tube diameter D, and the magnitude of the superfluid density ρ_s inside the grain boundary compared to the crystal density ρ_C. Assuming that the product $(a/e)(D/w)(\rho_C/\rho_s)$ is of order one, we found that the critical velocity in a grain boundary is of order 1.5 m s^{-1}. This would be very large for a classical liquid only about 1 nm thick but it is very close to what had been measured for a few atomic layers of liquid helium adsorbed on a wall [36].

However, during the writing of this article, we discovered that the liquid wets partially the line where the grain boundary meets the glass surface. This contact line is in fact a channel with a triangular cross-section. The order of magnitude of the channel width is 20 μm at 1 cm below the free liquid–solid level. We thus realized that there was another possible interpretation of our first experiment: the superflow could take place along the contact line of the grain boundaries with the glass. If right, this would mean that the critical velocity there is about 1 cm s^{-1}, a reasonable value for a 20 μm size channel. These channels should be superfluid at nearly the same temperature as the bulk liquid, and this is indeed what we observed. It is thus possible that the superfluid transition temperature inside a grain boundary is only a fraction of a Kelvin, as predicted by Pollet *et al*. [35].
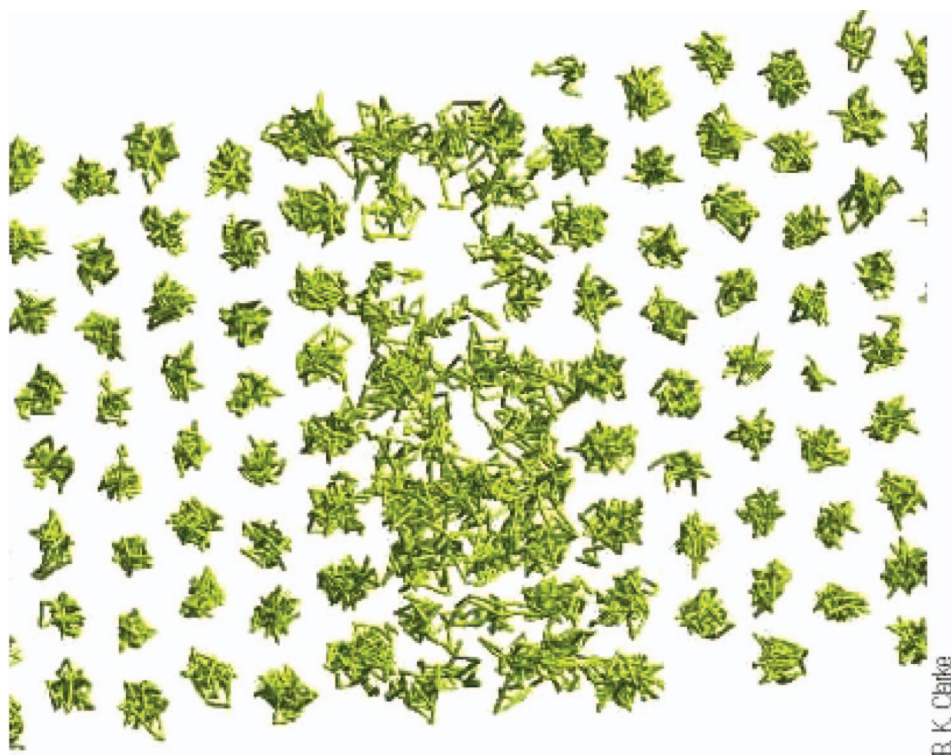

Figure 6. As shown by this numerical simulation by B. Clark and D. Ceperley [39], a grain boundary is a fluid layer a few atomic layers thick.

7. The end of the story?

Since the torsional oscillator measurements showed the importance of disorder, and since we found that mass flow is allowed by the presence of grain boundaries, we tried to interpret the experiment by Kim and Chan in terms of grain boundaries. However, as we shall see, it was not so easy. In my opinion, the whole issue is not yet settled.

The main problem is the amplitude of the supersolid density ρ_s which has been measured by Kim and Chan. They found values in the range from 0.3% to 1.5%. In order to achieve 1% with grain boundaries, and if one assumes that each grain boundary provides the equivalent of one atomic layer of superfluid matter, one needs typically one grain boundary every 100 atoms. With a two-dimensional set of grain boundaries, it could be one every 200 atoms. Since the interatomic spacing is 0.3 nm in ^{4}He crystals, the typical grain size should thus be 60 nm in the experiment by Kim and Chan. Obviously such a grain size looks very small. Measuring the density of grain boundaries in torsional experiments is not an easy task. We thus tried to grow crystals in a way similar to what is done in torsional oscillator experiments. Their crystals are
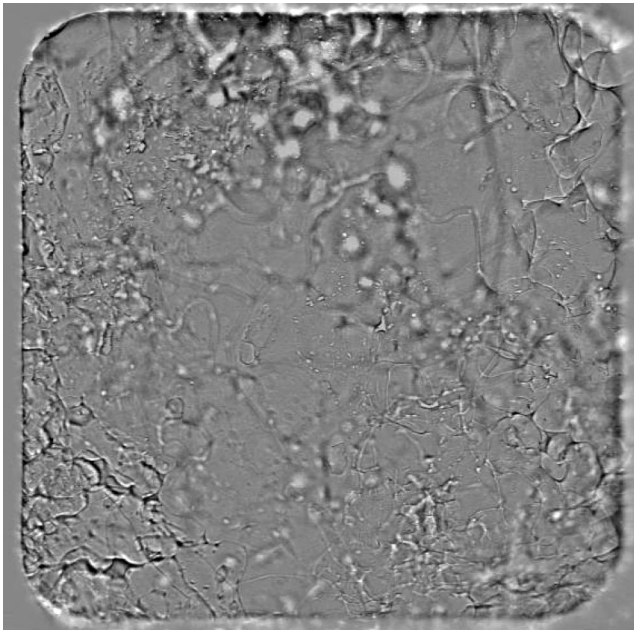

Figure 7. A photograph of a crystal grown at constant volume in a square cell. In order to reveal grain boundaries, the pressure has been lowered to the melting pressure after the completion of crystallization. The liquid wets the contact lines between the grain boundaries and the glass windows and these lines become visible. The density of grain boundaries is not high enough to explain supersolidity without something else.

grown at constant volume by the 'blocked capillary method'.

When doing this, we obtained crystals with many grain boundaries but whose typical spacing was visible (see figure 7). The grain boundary density was not as high as one every 60 nm. We had thus to find something else for an interpretation of torsional oscillator measurements. Part of the answer is suggested by new observations on the effect of annealing. Both Grigorev et al. [37] and Rittner and Reppy [38] have found that the pressure in their cells seemed to decrease by several bars after annealing. This is a large change which should be due to the existence of large inhomogeneities in density before annealing. It is possible that crystals grown at constant volume contain small liquid regions which crystallize during annealing. If this is true, then these liquid regions could provide the 1% superfluid density. The observation of a critical temperature T_c of order 100 mK would then be due to the superfluid transition inside the grain boundaries which would be the links between the liquid regions. At present, this is the only possible scenario I can imagine, but it is obviously rather speculative and needs to be carefully checked.

In summary, it seems to me that supersolidity has to do with quenched disorder and is not an intrinsic property of ^{4}He crystals, but before explaining everything in terms of superfluid grain boundaries and stress gradients, there are several experiments to do. They are in progress. I hope that we soon learn more on helium crystals, on their defects and on their quantum properties.

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