The optical exciton Aharonov-Bohm effect—i.e., an oscillatory component in the energy of optically active (bright) states—is investigated in nanorings. It is shown that a small effective electron mass, strong confinement of the electron, and high barrier for the hole, achieved, e.g., by an InAs nanoring embedded in an AlGaSb quantum well, are favorable for observing the optical exciton Aharonov-Bohm effect. The second derivative of the exciton energy with respect to the magnetic field is utilized to extract Aharonov-Bohm oscillations even for the lowest bright state unambiguously. A connection between the theories for infinitesimal narrow and finite width rings is established. Furthermore, the magnetization is compared to the persistent current, which oscillates periodically with the magnetic field and confirms thus the nontrivial (connected) topology of the wave function in the nanoring.

I. INTRODUCTION

The original Aharonov-Bohm effect (ABE) is found only for charged particles as a purely quantum-mechanical effect showing the important role of the vector potential. The ground-state energy of a charged particle oscillates with magnetic flux $\Phi_B = \pi r_0^2 B$ if the particle orbits in a ring around an infinitely long solenoid with radius $r_0$ where the magnetic field $B$ is concentrated. The oscillation period is given in units of the magnetic-flux quantum $h/e$. The ABE has been confirmed experimentally, e.g., in mesoscopic metal rings, and in doped semiconductor InAs/GaAs nanorings. The persistent current (PC) and the magnetization induced by an electron orbiting in mesoscopic metal and semiconductor rings have been measured also. Interacting electrons in the ring exhibits both the ABE and PC.

However, the exciton being a composite particle consisting of an electron and hole has zero total charge. Theoretical studies on the basis of a simplified model (zero width of the nanoring) have demonstrated that the optical exciton Aharonov-Bohm effect (X-ABE) is an oscillatory component in the energy of the optically active (bright) states. However, calculations including the finite width of the rings could not confirm these findings for the ground state. A recent calculation on two-dimensional annular lattices indicated that the X-ABE of the ground state for nanorings exists, but in this model the energy shift quadratic in the magnetic field was neglected. It is well known that for a ring radius $r$ much larger than the exciton Bohr radius $a_B$, the X-ABE is practically not observable. Several proposals have been made to overcome this limitation, such as applying an electric field to separate electrons and holes or different confinements for electron and hole. The effect of weak disorder or impurity scattering (in general losing the cylindrical symmetry) has been investigated with the result that optically nonactive (dark) states can become bright ones. In experiments, there are contradictory results: An ensemble of InP/GaAs type-II quantum dots has been studied in Ref. A theoretical explanation based on Ref. indicated some X-ABE oscillations in a single dot. However, in a very recent single-dot experiment on InP/GaAs quantum dots (grown under different conditions) no oscillations have been observed. Nevertheless, the X-ABE in nanorings has not been observed yet. But the ABE has been observed for charged excitons (complex of exciton and electron) in nanorings.

The aim of this work is to calculate the X-ABE, the PC, and the magnetization of the lowest optically active state in nanorings with finite width. A model which captures basic features of real materials, specifically different band alignments and strain, is used to investigate which material parameters are especially favorable for strong X-ABE (the persistent current or the magnetization). However, we stress that it is not the aim of this paper to model material properties with the most accurate description. For this reason some effects which may play an important role in selected materials, like piezoelectric fields, image charge effects, or even valence- and conduction-band mixing, will be neglected. After describing the theory in Sec. II, the results for the X-ABE are presented in Sec. III, followed by a discussion in Sec. IV. The persistent current and the magnetization are considered in Sec. V. The paper is summarized in Sec. VI.

II. THEORY

A. Exciton Hamiltonian

Excitons in a nanoring are described here within the envelope function formalism and applying the effective mass approximation (assuming parabolic bands). Including a constant $B$ field perpendicular to the ring plane, the Hamiltonian of a single exciton takes the following form:

$$
\hat{H} = \sum_{a=e,h} \left( \frac{1}{2m_{a,\perp}} \hat{p}_a - q_a A(r_a) \right)^2 + \frac{1}{2m_{a,\parallel}} \hat{p}_{z_a}^2 + U_a(z_a) + V_a(r_a) - \frac{e^2}{4 \pi \epsilon_0 \epsilon_s \sqrt{(r_z - r_0)^2 + (z - z_0)^2}},
$$

where $a$ denotes either electron ($e$) or hole ($h$), $m_{a,\parallel}$ is the in-plane and $m_{a,\perp}$ is the growth ($z\perp$) direction carrier effec-
tive mass (∥ is dropped in the following), $q_a$ is the charge ($q_e=-e$, $q_h=e$), $U_a(z_a)$ is the confinement potential in the growth direction, $V_{\parallel}(r_a)$ is the lateral confinement, and $\epsilon_a$ is the static dielectric constant. $r_a$ denotes the two-dimensional in-plane coordinates while $z_a$ is the coordinate in the growth direction. The Coulomb symmetric gauge of the vector potential is used: $A(r)=\frac{e}{r}\times r$. The spin degrees of freedom would bring in a term linear in the $B$ field (neglecting spin-orbit coupling):

$$\hat{H}^{\text{min}} = \sum_{a=e,h} g_a^* \mu_B B \sigma_a^z,$$

where $g_a^*$ are effective $g$ factors for electron and hole, $\mu_B = e\hbar/2m_e$ is the Bohr magneton ($m_e$ being the bare electron mass), and $\sigma^z$ is the Pauli spin matrix. Since electrons and holes have different $g$ factors, the spin term does not vanish for the exciton Hamiltonian. This spin-dependent part (Zeeman splitting) gives only a linear addition to the total exciton energy. For the sake of simplicity, it is not included in the following analysis. Only heavy-hole bright-exciton states (total angular momentum $J=1$) are considered in what follows.

Assuming that the nanoring is embedded in a narrow quantum well (schematically plotted in Fig. 1), a separation of the wave function for the in-plane and growth direction is adopted (single sublevel approximation),

$$\Phi(r_e,r_h,z_e,z_h) = \Psi(r_e,r_h)v_e(z_e)v_h(z_h),$$

where $v_e(z_e)$ are confinement wave functions. Furthermore, cylindrical symmetry is assumed for the lateral confinement. The single-exciton Hamiltonian, Eq. (1), is rewritten in polar coordinates in the following way:

$$\hat{H} = \sum_{a=e,h} \left[ -\frac{\hbar^2}{2m_a} \frac{\partial}{\partial r_a} \left( r_a \frac{\partial}{\partial r_a} \right) + \frac{1}{2m_a} r_a^2 \left( -i\hbar \frac{\partial}{\partial \phi_a} \right) - q_a eB r_a^2 \right]^{1/2} + V_a(r_a) + V_{\parallel}(r_a),$$

introducing the averaged Coulomb potential

$$V_C(r_e,r_h,\phi_e-\phi_h) = -\int dz \, dz_h \, \overline{\Psi^2(z_e)} \, \overline{\Psi^2(z_h)}$$

$$\times \frac{e^2}{4 \pi \epsilon_0 \epsilon_r \sqrt{r_e^2 + r_h^2 - 2r_e r_h \cos(\phi) + (z_e - z_h)^2}}.$$  

(5)

Due to the cylindrical symmetry of the Hamiltonian, Eq. (4), a transformation to new (Jacobi) angle coordinates is convenient,

$$\phi = \phi_e - \phi_h, \quad \Phi = \frac{1}{2}(\phi_e + \phi_h),$$

$$\frac{\partial}{\partial \phi_e} = \frac{\partial}{\partial \phi} + \frac{1}{2} \frac{\partial}{\partial \Phi}, \quad \frac{\partial}{\partial \phi_h} = -\frac{\partial}{\partial \phi} + \frac{1}{2} \frac{\partial}{\partial \Phi},$$

(7)

where $\phi$ (Φ) is the relative (average) angle. After transforming Eq. (4), the relation $[\hat{H} - i\hbar \frac{\partial}{\partial \Phi}] = 0$ can be easily verified. This enables the wave function factorization

$$\Psi(r_e,r_h,\phi,\Phi) = \psi(r_e,r_h,\phi) \frac{e^{im\Phi}}{\sqrt{2\pi}},$$

(8)

where $m$ is an integer quantum number characterizing the total angular momentum of the exciton envelope function. The exciton oscillator strength $f_\alpha$ of the state $\alpha$ is determined as follows:

$$f_\alpha = d_{\alpha} \int dr \Psi_\alpha^*(r,r),$$

(9)

where $d_{\alpha}$ is the interband dipole matrix element including the state-independent contribution of the confinement wave functions $v_e(z_e)$. The absorption spectrum (or optical density) is defined as

$$D(E) = \sum_\alpha |f_\alpha|^2 \delta(E - E_\alpha).$$

(10)

Introducing the factorization, Eq. (8), Eq. (9) simplifies to
f_a = d_c e \sqrt{2 \pi} \delta_{m,0} \int \psi_a(r_e, r_h, 0) r_e^2 dr_e. \quad (11)

Therefore, only states with \( m = 0 \) are optically active. Since the focus of this paper is to calculate optically active states, we set \( m = 0 \) throughout and end up with a Hamiltonian depending on three coordinates \( (r_e, r_h, \phi) \):

\[
\hat{H} = \sum_{a=e,h} \left\{ \frac{\hbar^2}{2m_a} \left[ -\frac{1}{r_a} \frac{\partial}{\partial r_a} \left( r_a \frac{\partial}{\partial r_a} \right) + \frac{i}{\hbar} \left( -\frac{\partial}{\partial \phi} + \frac{eB}{2\hbar} r_a^2 \right) \right] \right. \\
+ \left. V_a(r_a) \right\} + V_C(r_e, r_h, \phi). \quad (12)
\]

In order to obtain the eigenenergies \( E_a \) of the Hamiltonian the wave function is expanded into relative angular momentum eigenstates

\[
\psi_a(r_e, r_h, \phi) = \sum_i u_i(r_e, r_h) e^{i \phi} \frac{1}{\sqrt{2\pi}}. \quad (13)
\]

The Hamiltonian matrix for the functions \( u_i(r_e, r_h) \) can be derived straightforwardly:

\[
\hat{H}^{ii'}(r_e, r_h) = \delta_{ii'} \sum_{a=e,h} \left\{ \frac{\hbar^2}{2m_a} \left[ -\frac{1}{r_a} \frac{\partial}{\partial r_a} \left( r_a \frac{\partial}{\partial r_a} \right) \right] \right. \\
+ \left. \frac{1}{r_a^2} \left( l + \frac{eB}{2\hbar} \right)^2 \right\} + V_a(r_a) + V_C^{ii'}(r_e, r_h). \quad (14)
\]

Kinetic and confinement terms (curly braces) are diagonal in \( l \). The Coulomb potential is nondiagonal in \( l \) and given by

\[
V_C^{ii'}(r_e, r_h) = \frac{1}{2\pi} \int_0^{2\pi} V_C(r_e, r_h, \phi) \cos(k\phi) d\phi. \quad (15)
\]

We note that the diamagnetic shift of state \( a \) is usually defined in literature as \(^{31}\)

\[
\Delta E_{a,\phi}(B) = E_{a}(B) - E_{a}(0). \quad (16)
\]

Our goal is to extract from the total diamagnetic shift \( \Delta E_{a}(B) \) the oscillatory component.

In the following, the wave function of the state \( a \) is analyzed studying the correlated one-particle densities:

\[
n_a^{(e)}(r_e, \phi) = r_e \int d r_h^2 |\psi_a(r_e, r_h, \phi)|^2, \quad (17)
\]

\[
n_a^{(h)}(r_h, \phi) = r_h \int d r_e^2 |\psi_a(r_e, r_h, \phi)|^2, \quad (18)
\]

for which the angular position of the second particle is fixed at, say, \( \phi = 0 \). Another possibility would be to fix the \( r_e \) coordinate instead of integrating over \( r_e \). In the case of strong confinement in the ring both approaches are equivalent. The numerical solution is performed by the Lanczos method, giving only a few lowest eigenstates of the Hamiltonian, Eq. (12). The absorption spectrum, Eq. (10), is calculated by the time propagation method for the optical interband polarization.\(^{32}\)

B. Persistent current and magnetization

Even though the exciton is a neutral particle, it can possess a current at a finite \( B \) field: electrons and holes orbit in the nanoring under the \( B \) field in opposite directions, and since they have opposite signs of their charges, their current contributions do add. The exciton persistent current is closely related to the ABE as already pointed out in the literature.\(^{12,22,33}\) The one-particle current density operator at position \( \mathbf{r} \) is defined as

\[
\hat{J}_a(r) = \frac{q_a}{2m_a} \left[ (\hat{\mathbf{p}}_a - q_a A(r_a)) \delta(\mathbf{r} - \mathbf{r}_a) \right. \\
+ \left. \delta(\mathbf{r} - \mathbf{r}_a) [\hat{\mathbf{p}}_a - q_a A(r_a)] \right]. \quad (19)
\]

The expectation value of the radial current \( \hat{J}_{a,r} \) is nonzero only for continuum states and will not be discussed further. In the present cylindrical symmetry, the azimuthal current \( \hat{J}_{a,\phi}(r, \phi) \) takes the form

\[
\hat{J}_{a,\phi}(r, \phi) = \frac{q_a}{2m_a} \left[ \left( -\frac{i\hbar}{r_a} \frac{\partial}{\partial \phi_a} - \frac{q_a B r_a^2}{2} \right) \delta(\mathbf{r}_a - \mathbf{r}) \right. \\
+ \left. \delta(\mathbf{r}_a - \mathbf{r}) \left( -\frac{i\hbar}{r_a} \frac{\partial}{\partial \phi_a} - \frac{q_a B r_a^2}{2} \right) \right]. \quad (20)
\]

The total exciton current consists of the electron and hole ones, which have to be added and integrated over the cross section of the nanoring,

\[
I_a(\phi) = \int dr(\alpha|\hat{J}_{e,\phi}(r, \phi) + \hat{J}_{h,\phi}(r, \phi)|\alpha). \quad (21)
\]

In accordance with Kirchhoff’s laws of current conservation \( I_a(\phi) \) is independent of the angle \( \phi \). The one-particle magnetization operator is defined as

\[
\hat{M}_a(r) = \frac{1}{2} \mathbf{r} \times \hat{J}_a(r). \quad (22)
\]

The only nonzero expectation value of the magnetization integrated over all space is directed along \( z \) and has the magnitude

\[
M_a = \pi \int drr^2(\alpha|\hat{J}_{e,\phi}(r, \phi) + \hat{J}_{h,\phi}(r, \phi)|\alpha). \quad (23)
\]

With the expansion, Eq. (13), the persistent current and the magnetization can be written explicitly as

\[
I_a = \frac{e\hbar}{2\pi} \sum_i \int drr \left( \frac{1}{r^2} - \frac{1}{2\lambda^2} \right) \\
\times \int dr' r' \left( \frac{u_{i,a}^2(r, r')}{m_e} + \frac{u_{i,a}^2(r', r)}{m_h} \right), \quad (24)
\]
\[ M_a = \frac{e\hbar}{2} \sum_i \int d\tau \left( 1 - \frac{r^2}{2\lambda^2} \right) \times \int d\tau' r' \left( \frac{u^2(r, r')}{m_e} + \frac{u^2(r', r)}{m_h} \right). \]  

(25)

Comparing Eqs. (24) and (25), the difference is an additional factor \( \pi r^2 \) in the integrand of the magnetization. The expression for the magnetization can also be derived using the Hellmann-Feynman theorem
\[ M_a = -\frac{dE_a(B)}{dB}. \]  

(26)

C. Infinitesimal narrow ring (simplified model)

In the limiting case of strong electron and hole confinement which can be achieved either by combining materials with appropriate band alignments or by the inclusion of strain, the wave function can be further factorized:
\[ \psi(r_e, r_h, \phi) = f_a(r_a) \chi(\phi), \]  

(27)

where \( f_a(r_a) \) are one-particle confinement wave functions in the radial direction centered at \( R_a \). The eigenfunction \( \chi(\phi) \) can be expanded as in the previous expansion, Eq. (13):
\[ \chi_a(\phi) = \sum_i u_{i, a} e^{i\phi_i}/\sqrt{2\pi}. \]  

(28)

After averaging the Hamiltonian, Eq. (14), with functions \( f_a(r_a) \) one gets a new Hamilton matrix
\[ \hat{H}_{ii'} = \delta_{ii'} \sum_{a=e, h} \left[ \frac{\hbar^2}{2m_a R_a^2} \left( l + \frac{eB}{2\hbar} R_a^2 \right)^2 \right] + \tilde{V}_{C}'(R_e, R_h) \]  

\[ = \delta_{ii'} \left[ \frac{\hbar^2}{2\mu X X} \left( l + \frac{eB}{2\hbar} R_X^2 \right)^2 + \Delta E^{(2)}(B) \right] + \tilde{V}_{C}'(R_e, R_h), \]  

(29)

where \( \mu_X = m_e m_h / (m_e + m_h) \) is the reduced exciton mass. The energies of the radial motion are omitted. In forming a complete square from the electron and hole kinetic terms, Eq. (30), an effective ring radius for the exciton,
\[ R_X^2 = \frac{R_e^2 R_h^2 (m_e + m_h)}{m_e R_e^2 + m_h R_h^2}, \]  

(31)

and a residual energy
\[ \Delta E^{(2)}(B) = \frac{e^2 B^2 (R_e^2 - R_h^2)^2}{8 m_e R_e^2 + m_h R_h^2}, \]  

(32)

appear.

We note that this Hamiltonian has been intensively studied since its introduction. The further simplification of the Coulomb potential to \( \tilde{V}_C(R_e, R_h) = U_C \) (contact potential) enables an analytical solution. The case of different electron and hole radii has been studied also.

The total energy can be written as
\[ E_a(B) = E_a(0) + \Delta E^{(1)} + \Delta E^{(2)}(B). \]  

(33)

From the structure of Eq. (30) it is clear that \( \Delta E^{(1)} \) is a strictly periodic function of \( B \) with the period
\[ B_p = \frac{2h}{e R_X^2}, \]  

(34)

and is called the oscillatory component.

The persistent current induced by an exciton in state \( \alpha \) [Eq. (24)] can be also obtained from a version of the Hellmann-Feynman theorem after introducing the one-particle flux
\[ \Phi_{B,a} = \pi R_X^2 B : \]  

\[ I_a = -\frac{\partial E_a(\Phi_{B,e}, \Phi_{B,h})}{\partial \Phi_{B,e}} - \frac{\partial E_a(\Phi_{B,e}, \Phi_{B,h})}{\partial \Phi_{B,h}}. \]  

(35)

In the present case, however, this would call for a calculation of the problem in dependence on two different (fictitious) \( B \) fields in \( E_a(\Phi_{B,e}, \Phi_{B,h}) \). From the Hamiltonian, Eq. (30), the exciton flux can be defined as
\[ \Phi_{B,X} = \pi R_X^2 B, \]  

(36)

and the exciton energy can be regarded as a function of the exciton flux \( E_a(\Phi_{B,x}) \). It turns out that the exciton PC calculated from the definition, Eq. (35), is equivalent to the derivative of the oscillatory component of the exciton energy only.
\[ I_a = -\frac{1}{\pi R_X^2} \frac{d\Delta E^{(1)}(B)}{dB}. \]  

(37)

This means that the term quadratic in \( B(\Delta E^{(2)}(B)) \) does not contribute to the PC. From the general definition of the magnetization, Eq. (26), a relation between the magnetization and the PC can be found easily,
\[ M_a = \pi R_X^2 I_a = -\frac{d\Delta E^{(2)}(B)}{dB}. \]  

(38)

The second term is the intrinsic magnetization originating from the inner electron and hole motion in the exciton while the first one, being proportional to the PC, is related to the nontrivial (connected) topology of the wave function. Only in the case of identical electron and hole radii, \( R_e = R_h \), are the PC and magnetization proportional, since the term \( \Delta E^{(2)}(B) \) is absent.

In the following, we will refer to the finite-width (zero-width) ring equivalently as to the full (simplified) model.

D. Strain

The strain plays an important role in some materials and should be included at least in a first approximation. Recently a simple but for our purpose well-adjus ted approach of isotropic elasticity in nanostructures has been developed. Assuming barrier and well material to be identical in their elastic properties, the strain tensor \( \epsilon_{ij} \) reads
The mismatch between ring lattice constant \( H_{9261} \) method for observing oscillations: The second derivative of rings represents an open question. We propose the following since its proper confirmation in finite width nano-optical exciton aharonov-bohm effect, where \( S \) lattice constant \( H_{20849} \) potentials and \( E_{e} \) Eq. \( H_{20849} \) zero Coulomb potential, Eq. \( H_{20849} \) confinement types. We investigate \( H_{9280} \) well. The electron and heavy-hole band edges are modified due to the strain in the simplest form

\[
\epsilon_{i}(r) = -\epsilon_{s}\theta(r \in \text{ring}) - \frac{\epsilon_{s} 1 + \nu}{4\pi 1 - \nu} \int_{\text{ring}} \frac{(x_{i} - x_{i}')}{[r - r']^{3}},
\]

where \( S' \) is the nanoring surface over which is integrated, \( \nu \) is the Poisson ratio, \( \epsilon_{s} = (\lambda_{\text{ring}} - \lambda_{\text{well}}) / \lambda_{\text{well}} \) is the relative lattice mismatch between ring lattice constant \( \lambda_{\text{ring}} \) and well lattice constant \( \lambda_{\text{well}} \). The electron and heavy-hole band edges are schematically shown in Fig. 1. The effective particle is found predominantly. The effective masses are chosen according to the material in which the \( x_{e} \) and hole are confined together, parameters are summarized in Tables I and II. The effective mass is found predominantly.

In the investigation of the X-ABE we concentrate on the type-I nanorings, where the electron and hole are confined together, (ii) type-II-A nanorings, where the electron is confined in the ring and the hole has a ringlike barrier, and (iii) type-II-B nanorings, where the hole is confined in the ring and the electron has a ringlike barrier (schematically shown in Fig. 1). The well and ring material parameters are summarized in Tables I and II. The effective masses are chosen according to the material in which the particle is found predominantly.

In the investigation of the X-ABE we concentrate on the optically active state with the lowest eigenenergy \( E_{e} \) in what follows, since its proper confirmation in finite width nanorings represents an open question. We propose the following method for observing oscillations: The second derivative of the energy with respect to the \( B \) field. It has already been mentioned in the Introduction and is shown below that the oscillations of the excited states are much easier visible due to their higher kinetic energy, leading to much larger exciton extension.

We have chosen \( B \)-field strengths up to \( B = 25 \) T which can be easily achieved in experiment.

A. Type-I ring: GaAs/AlGaAs

The choice of GaAs/AlGaAs for a type-I structure is rather straightforward since it is the most frequently investigated direct semiconductor. The strain can be safely neglected in this structure due to the small lattice mismatch (in contrast to previously investigated self-assembled InAs/GaAs nanorings\(^{39}\)). There is even a newly developed technique which allows one to grow concentric nanorings.\(^{44}\) The structure investigated here consists of a \( \text{Al}_{0.23}\text{Ga}_{0.77}\text{As} \) 4-nm-wide quantum well between \( \text{Al}_{0.3}\text{Ga}_{0.7}\text{As} \) barriers. A nanoring of pure GaAs is placed inside the \( \text{Al}_{0.23}\text{Ga}_{0.77}\text{As} \) well.

As an example we discuss the absorption spectrum, Eq. \( 10 \), of an GaAs/AlGaAs nanoring as plotted in Fig. 2(a) where the oscillations of the excited states are indeed clearly visible, while the ground state shows only a smooth and monotonic energy shift. Before discussing the properties of the lowest bright state we focus our attention on one interesting feature of excited states: namely, the anticrossing marked by a circle in Fig. 2. At a first glance, the absorption spectrum resembles the result of the simplified model.\(^{13}\) The first three lowest bright states can also be found in the sim-

**TABLE II. Chemical band edges, Eq. \( 40 \), in meV, relative lattice mismatch \( \epsilon_{s} \), and static dielectric constant \( \epsilon_{s} \).**

<table>
<thead>
<tr>
<th>Material</th>
<th>( E_{e} )</th>
<th>( E_{h} )</th>
<th>( \epsilon_{s} ) (%)</th>
<th>( \epsilon_{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs/AlGaAs</td>
<td>−257</td>
<td>−110</td>
<td>0</td>
<td>12.5</td>
</tr>
<tr>
<td>InP/GaInP</td>
<td>−600</td>
<td>50</td>
<td>3.81</td>
<td>12.6</td>
</tr>
<tr>
<td>InAs/AlGaaSb</td>
<td>−1673</td>
<td>332</td>
<td>−1.00</td>
<td>15(^{a})</td>
</tr>
<tr>
<td>GaSb/GaAs</td>
<td>−63</td>
<td>−770</td>
<td>7.83</td>
<td>12.5</td>
</tr>
</tbody>
</table>

\(^{a}\) Taken from Ref. 42.
plified model as plotted in Figs. 2(b)–2(d) (being even, even and odd with respect to \( \phi \) at \( B=0 \) T). Only the fourth state [Fig. 2(e)] cannot be found in the simplified model since the hole sits in its first excited radial state, which is absent in a zero width ring. Its overlap with the electron part and consequently the oscillator strength are, however, tiny. Nevertheless, this even state manifests itself strongly by the anticrossing with the next even state at around \( B=13 \) T. This kind of anticrossing, even though somewhat marginal, goes beyond the description of the simplified model. From now on, let us concentrate on the lowest bright state.

Up to now there has not been any clear evidence of oscillations of the lowest bright (ground) state for finite width nanorings. The problem becomes evident looking at Fig. 3(a), where on the first glance the only dependence of the energy on the \( B \) field is the smooth and monotonic increase. Although for nanorings of finite width a separation of the diamagnetic shift like Eq. (33) is not possible in a strict sense, we will understand in the following how \( \Delta E^{(1)}_a \) as the oscillating part and \( \Delta E^{(2)}_a \) as the smooth monotonic part. The behavior of the exciton ground-state energy in the limit \( B \to 0 \) has been studied in Ref. 45, finding a nontrivial dependence on the one-particle confinement and exciton relative motion. In the present case, the strong electron (hole) ring confinement fixes the electron (hole) radial position \( r_{e(h)} \) and the strong Coulomb interaction fixes the relative distance \( r \), which means that the quadratic dependence on \( B \) and consequently its contribution to the second derivative are almost constant with the \( B \) field. This enables to extract the second—oscillatory—component from the second derivative as seen in Fig. 3(b). The strong dependence of the oscillations amplitude on the ring radius is remarkable. The expectation values of \( R^2 = \langle r_{e(h)}^2 \rangle \) at \( B=0 \) T from the full solution were used as input parameters in the simplified model. A comparison of the simplified with the full model shows good agreement for the period and the amplitude of the oscillations and its strong dependence on the ring radius as well. The main difference is the absence of the term \( \Delta E^{(2)}_x \) in the

FIG. 2. Absorption spectrum (a) of a GaAs/AlGaAs nanoring with radii \( r_1=4 \) nm and \( r_2=12 \) nm. The lines are Gauss broadened with \( \sigma=1 \) meV, and the oscillator strength (for the ground state divided by 10) is given in linear gray scale (steplike features are artifacts of the interpolation). The circle focuses on the specific anticrossing (see text). The correlated hole densities at \( B=2 \) T are given for the first four lowest bright states (b)–(e). The black rectangle indicates the fixed electron position.

FIG. 3. (Color online) The \( B \)-field dependence of the lowest bright (ground) state energy (a) and its second derivative (b) in type-I GaAs/AlGaAs nanorings: solid line, \( r_1=4 \) nm, \( r_2=12 \) nm, and dashed line, \( r_1=8 \) nm, \( r_2=16 \) nm. The full calculation (black) is compared to results for infinitesimal narrow rings, Eq. (30) (red). The periods of the oscillations, Eq. (34), are \( B_P=20.8 \) T (solid line) and \( B_P=9.2 \) T (dashed line). Projected hole density \( n^{(h)} \) according to Eq. (18) at \( B=0 \) T (c). The ring boundaries are shown as dashed circles.
simplified model. It is the finite radial extension of the exciton relative wave function [plotted in Fig. 3(c)] which gives a nonzero contribution to this energy. After having compared results of both models in this case, further we will discuss only the full solution. Note, however, that the period, Eq. (34), gives generally a good estimate.

Let us direct our attention to type-II systems where more pronounced effects are expected.

B. Type II-A: InP/GaInP

InP/GaInP self-assembled quantum dots are of type II-A and have been investigated since many years both theoretically and experimentally (see Ref. 46 and references therein). Possibly, rings may be grown as well, e.g., using the same procedure as for InAs/GaAs (Ref. 47) or GaAs/AlGaAs (Ref. 44) nanorings. We have investigated a structure consisting of a 4-nm-wide Ga0.51In0.49P quantum well between AlAs barriers. The nanoring is pure InP. Such a structure guarantees that the hole is always found around the ring (in the xy plane) and not above or below the ring [in the growth (z) direction]. This is not a necessary condition for the X-ABE. The situation where the hole (electron) is found above or below is also of interest. This goes beyond the scope of this paper since we would not be able to take advantage of the z separation. The strain plays an important role in this material,39 as is clearly shown in Fig. 4(a).

Since the deep minimum of the hole potential is formed at the inner edge of the ring [Fig. 4(a)], the hole is found there. Such a state is named hole-in [depicted in Fig. 5(c)]. The effective electron-hole separation is thus decreased with respect to the strain-free case. The state hole-in is the ground state for any ring radius. Excluding composition changes, the height of the ringlike barrier for the hole can decrease by changing the well width in the z direction. For high B fields, a transition from type II to type I may occur due to the enhanced penetration of the hole wave function into the ring. This has been already predicted for quantum dots in Ref. 39.

The energy of the lowest bright states as a function of B field is plotted in Fig. 5(a) Again, no evidence of oscillations is seen by the naked eye. The analysis of the second derivative [Fig. 5(b)] reveals that (i) the amplitude of the oscillation is increased compared to type I (as expected) and (ii) the period of the oscillation is increased as well since the hole samples a smaller magnetic flux [see Eq. (34)] compared to type I.

C. Type II-A: InAs/AlGaSb

The InAs/AlGaSb system has several advantages for observing the X-ABE in type-II-A systems compared to InP/GaInP as will be discussed below. This system is less known compared to GaAs/AlAs or InP/GaInP, but as a quantum well structure well understood and used (see Ref. 48 and references therein). Recently, InAs quantum dots on an AlGaSb substrate have been grown.49 The fact that Al0.65Ga0.35Sb is an indirect semiconductor50 is of less impor-

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**FIG. 5.** The B-field dependence of the lowest bright state energy (a) and its second derivative (b). Solid line, \( r_1=4 \) nm, \( r_2=12 \) nm, and dashed line, \( r_1=8 \) nm, \( r_2=16 \) nm, InP/GaInP nanorings. The solid line in (a) is shifted by 20 meV for comparison. Projected hole density \( n^{(b)} \) according to Eq. (18) at \( B=0 \) T (c). The ring boundaries are shown as dashed circles. The periods of the oscillations, Eq. (34), are \( B_P=40.9 \) T (solid line) and \( B_P=11.8 \) T (dashed line).
tance since the electron is found predominantly in InAs, which means that the approximation taking into account only the \( \Gamma \) point is sufficient. A problem is the small difference (0.083 eV) between the conduction-band edge in InAs and the valence-band edge in Al\(_{0.6}\)Ga\(_{0.4}\)Sb.\(^{40}\) The applicability of the effective-mass approximation is questionable here.\(^{51}\) Nevertheless, we believe as a first approximation\(^ {42}\) it can be adopted. The investigated structure consists of a Al\(_{0.6}\)Ga\(_{0.4}\)Sb 4-nm-wide quantum well between AlSb barriers. A InAs nanoring is placed in the Al\(_{0.6}\)Ga\(_{0.4}\)Sb well. Even though the lattice mismatch between InAs and AlGaSb is small,\(^ {1\%}\) our calculation includes strain [see Fig. 4(b)].

The influence of the strain on the hole potential for InAs/AlGaSb is shown in Fig. 4. Compared to InP/GaInP, there are striking differences. The effect of strain is much smaller due to the much smaller lattice mismatch and the sign of the strain contribution is opposite. Instead of compression in the ring as for InP/GaInP (enlarging the band gap), there is dilatation in the case of InAs/AlGaSb which lowers the band gap. This leads to the repulsion of the hole from the ring and thus to a weakening of the Coulomb interaction. In contrast to InP/GaInP, the minimum of the hole potential is found outside of the ring [Fig. 4(b)] for any ring radius. If the hole is found outside the ring, such state is called hole-out [depicted in Fig. 6(c)]. The difference between the potential value in the middle and outside the ring decreases with increasing inner ring radius.

As stated above, due to its material properties a large oscillation amplitude is found here, as seen in Figs. 6(a) and 6(b). Even without any further analysis, the lowest bright state, hole-out, shows clear oscillations. Note a change of the scale by a factor of 10 in Fig. 6(b) compared to Fig. 4(b). In both cases kinks in \( E(B) \) (sharp minima in \( d^2E/dB^2 \)) resemble the one-particle ABE and are consequence of the weak Coulomb interaction.

An interesting effect is found in larger rings: namely, a transition from hole-in to hole-out. Depending on the ring geometry one of them is the lowest bright state and the other one the second lowest. The strain profile favors the state hole-out. On the other hand, the Coulomb interaction prefers the state hole-in. As the strain profile in the middle and outside of the ring becomes similar for larger rings, the Coulomb interaction dominates and the state hole-in becomes the lowest bright state. This situation is demonstrated in Fig. 7(b). The lowest bright state changes with increasing \( B \) field: from hole-in [Fig. 7(b)] to hole-out [Figs. 7(c) and 7(d)]. The state hole-in has a larger energy shift \( \Delta E \)\(^ {2}\), which can be verified by calculating the effective hole radii \( (r_B^2) \) and checking Eq. (32). The transition occurs at around \( B=1.5 \) T. Since we always follow the lowest bright state the second derivative shows a sharp peak at the transition [Fig. 7(a)]. We note that the small overlap of the hole-in and hole-out wave functions does not allow to distinguish between level crossing and anticrossing, at least within our numerical precision. Due to the large radius of the ring, the oscillation period is small [according to Eq. (34) \( B_p=1.9 \) T]. The decay of the oscillation amplitude is due to a decrease in exciton Bohr radius with \( B \) field [compare Figs. 7(c) and 7(d)].

**FIG. 6.** The \( B \)-field dependence of the lowest bright state energy (a) and its second derivative (b). Solid line, \( r_1=4 \) nm, \( r_2=12 \) nm, and dashed line, \( r_1=8 \) nm, \( r_2=16 \) nm, InAs/AlGaSb nanorings. Projected hole density \( n_{\text{b}} \) according to Eq. (18) at \( B=0 \) T (c). The ring boundaries are shown as dashed circles. The period of the oscillations, Eq. (34), are \( B_p=19.9 \) T (solid) and \( B_p=8.8 \) T (dashed line).

**FIG. 7.** The second derivative with respect to the \( B \) field of the lowest bright state energy (a) of the InAs/AlGaSb nanoring with radii \( r_1=22 \) nm and \( r_2=30 \) nm. Projected hole density \( n_{\text{b}} \) according to Eq. (18) at \( B=0 \) T (b), \( B=10 \) T (c), and \( B=20 \) T (d). The ring boundaries are shown as dashed circles. The period of the oscillations, Eq. (34), is \( B_p=1.9 \) T.
D. Type II-B: GaSb/GaAs

GaSb/GaAs self-assembled quantum dots of type II-B have attracted a certain interest recently (see Ref. 52 and references therein). The strain plays a very important role in these structures and modifies significantly the conduction- and valence-band energies: The strain-free offsets [Eq. (40)] \( E_c \) = 0.063 eV and \( E_h \) = −0.77 eV are modified to \( E_c \) = 0.65 eV and \( E_h \) = −0.86 (minimum). The substantial change of the electron potential is shown in Fig. 8. These results are comparable with those in Ref. 53. The investigated structure consists of a GaAs 4-nm-wide quantum well between Al\(_{0.3}\)Ga\(_{0.7}\)As barriers, with a GaSb nanoring placed in the well.

The increase of the lowest bright state energy by 40 meV from \( B \) = 0 T to \( B \) = 25 T [Fig. 9(a)] is large compared to all previous values and again no clue of oscillation is seen. In the second derivative [Fig. 9(b)], a sharp initial decay is revealed. The origin of this decay can be understood studying the wave function. The hole is strongly confined in this system and the electron potential has a high ring like barrier as mentioned above. The correlated electron density plotted in Fig. 9(c) shows that the shallow Coulomb potential localizes the electron part of the wave functions only weakly. The electron is very sensitive to the \( B \) field. Due to the quadratic term \( \Delta E^{(2)} \), the electron is forced to move quickly towards the hole (i.e., to the nanoring) with increasing \( B \) field. This behavior leads to the initial decay in the second derivative [Fig. 9(b)]. Later on, the wave function stabilizes and oscillations appear [Fig. 9(b)]. Their amplitude is comparable to values found in InP/GaInP. The advantage of the GaSb/GaAs system is the large Bohr radius due to the large electron-hole separation. On the contrary, the disadvantage is the sensitivity of the electron to the \( B \) field due to its small mass and shallow confinement.

IV. DISCUSSION OF X-ABE

Now, we compare and discuss the results of the previous section and conclude which material combination is preferential for X-ABE.

Our results show unambiguously that a weakening of the Coulomb interaction increases the oscillation amplitudes, which has already been shown for the simplified model.\(^{14,22}\) The reason is that electrons and holes can sample the entire ring more easily if the exciton is weakly bound (larger exciton Bohr radius) and the wave function can acquire the necessary ring topology. The mutual confinement of electrons and holes (type I) has turned out to be inferior to the systems where electron and hole are separated by the conduction- and valence-band alignments in the \( xy \) plane (type II). One unwanted consequence of the spatial electron-hole separation is that the lowest bright state is not anymore the ground state for larger \( B \) fields\(^{15,22,35}\) (in contrast to type I). This may result, e.g., in losses of photoluminescence intensity if some nonradiative decay channels are present. A calculation of these kinetic effects goes, however, beyond the scope of this paper.

Comparing the results for different material systems, we find that for a large amplitude of X-ABE oscillations the ideal structure is of type II-A with the following properties: light electron mass, strong electron confinement, and high barrier for holes. These criteria can be discussed qualitatively: (i) The light electron mass leads to a larger Bohr radius and higher probability of the particle to sample the whole nanoring. (ii) Strong electron confinement is needed in order to force the light electron to orbit around the ring. (iii) A high barrier for holes is necessary for a “good” type-II nanoring, thus avoiding the penetration of the wave function into the ring. The necessity of a high (deep) potential of the lighter particle has already been pointed out.\(^{35}\) The material which matches these criteria best is InAs/AlGaSb in our
FIG. 10. Magnetization (a)–(d) and persistent current (e)–(h) for nanorings with radii \(r_1=4\) nm, \(r_2=12\) nm (solid line) and \(r_1=8\) nm, \(r_2=16\) nm (dashed line) of the lowest bright state. Materials: GaAs/AlGaAs (a), (e), InP/GaNP (b), (f), InAs/AlGaSb (c), (g), and GaSb/GaAs (d), (h). The scaling factor between the PC and the magnetization is held constant in all cases.

This system clearly deserves further investigations, both theoretically and experimentally.

VI. PERSISTENT CURRENT AND MAGNETIZATION

After having examined the X-ABE in different materials, we investigate now the persistent currents and the magnetization in a more compact way.

The PC, Eq. (24), and the magnetization, Eq. (25), can be measured under special conditions: (i) The exciton should be excited into the optically active state (in our case always the lowest one). (ii) The excitation power should be sufficient in order to give a measurable signal but small enough to avoid exciton-exciton interaction. We assume one exciton per nanoring in the following, which corresponds to extremely strong excitation. A more realistic value of the excitation would reduce the scales in Fig. 10 accordingly. The state-of-the-art experimental technique [superconducting quantum interference device (SQUID)] enables one to measure the magnetization directly.\(^7\) The measurement of the current requires additional contacts on the nanoring which may complicate the already difficult assembling of the nanorings even more. A measurement of either the persistent current or the magnetization in the nanoring is an extremely challenging task.

First, the magnetization of the exciton in the ring with finite width [Figs. 10(a)–10(d)] can be divided roughly also into two contributions according to the analogy with the simplified model, Eq. (38): The first (oscillatory) part comes from the orbiting of electrons and holes around the nanoring while the second monotonic part (nearly linear in \(B\)) is related to the inner exciton motion where electrons and holes orbit around each other. The weight of each contribution depends on the wave function topology. The dominance of the first contribution is seen only in the case of a weak Coulomb interaction where the magnetization has even negative values—i.e., pointing into the opposite direction of the \(B\) field itself. Such an effect could be interesting for further applications: namely, a sign switch of the optically induced coherent magnetization by the \(B\) field. Unfortunately, this effect is rather weak. In all other cases both parts contribute with different weights. The linear component is found also for excitons in quantum wells or dots.

Second, the exciton PC shown in Figs. 10(e)–10(h) exhibits periodic oscillations for each ring geometry and for all materials, even though the oscillation amplitude may be very small [as, e.g., in Fig. 10(d)]. The period and relative amplitude of the oscillations agree well with those of the second derivative of the energy with respect to the magnetic field. Furthermore, the results confirm relation Eq. (37) qualitatively also for finite-width nanorings since they indeed remind one of the first derivative of the oscillatory component of the exciton energy. A quantitative comparison is in general not possible since the smooth (nonoscillatory) component of the diamagnetic shift cannot be extracted unambiguously. From the theoretical point of view it turns out that a measurement of the PC would give a more direct information on the nontrivial ring topology of the wave function.

VI. SUMMARY

In summary, we have investigated the optical exciton Aharonov-Bohm effect, persistent current, and magnetization in nanorings. We have discussed the differences between a simplified and the full model. In the simplified model, the
smooth monotonic part of the energy is small or even missing. We have focused on the observability of the X-ABE in type-I and -II nanorings which can be improved by taking appropriate materials, at least, by one order of magnitude. We have discussed in detail GaAs/AlGaAs (type I), InP/GaInP and InAs/AlGaSb (type II-A), and GaSb/GaAs (type II-B) nanorings and found that also in large InAs/AlGaSb nanorings the X-ABE can be observed. We have shown that a hole-in–hole-out transition occurs in these nanorings. The persistent current is found to be proportional to the first derivative of the oscillatory component of the exciton energy and thus has a close relation to the X-ABE. In the case of the magnetization we have demonstrated that oscillations are superimposed on the linear component related to the inner exciton motion, being independent of the ring topology of the wave function.

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46 J. Persson, U. Hakanson, M. K.-J. Johansson, L. Samuelson, and...