NAME:

MIDTERM EXAM

PHYS852 Quantum Mechanics II, Spring 2009

1. [50 pts] Consider a system of two spin-1/2 particles, with corresponding spin operators, $\vec{S}_1$, and $\vec{S}_2$. They do not interact with each other, but they both interact with a spin-1 particle, with spin operator $\vec{I}$. Let the bare Hamiltonian for the system be

$$H_0 = 2\frac{\omega}{\hbar}\left[\vec{S}_1 \cdot \vec{I} + \vec{S}_2 \cdot \vec{I}\right]$$

   a.) [10 pts] Let $\vec{S} = \vec{S}_1 + \vec{S}_2$. What are the allowed values of the quantum number s? For each s what are the possible m_s values?

   b.) [10 pts] Let the total angular momentum operator be $\vec{F} = \vec{S} + \vec{I}$. For each possible s-value, give the allowed values of the quantum number f. For each state with well-defined s and f, list the possible m_f values.

   c.) [10 pts] What are the eigenvalues of H_0? For each eigenvalue, give the degeneracy factor, and list the corresponding degenerate states.

   d.) [10 pts] Now assume a magnetic field, $\vec{B} = B_0\vec{e}_z$, is applied. Assuming both spin-1/2 particles have charge q and mass m, and neglecting the magnetic moment of the spin-1 particle, give the operator which describes the interaction between the system and the field?

   e.) [10 pts + possible extra credit] In the limit of a very weak (but non-zero) field, describe as best you can what happens to each of the bare energy levels when the field is applied.

2. [50 pts] An exciton is a hydrogen-like bound-state of a conduction electron and a hole inside a semi-conductor. Let $|g\rangle$ indicate the state with no excitons present, and let $|k\rangle$ indicate a one-exciton state with momentum $\hbar k$.

   Single-photon absorption can create excitons from the $|g\rangle$ state. In the proper rotating frame, the effective Hamiltonian for a laser-driven semi-conductor is

$$H = \hbar|\Delta| \sum_k |k\rangle\langle k| + \hbar\frac{\Omega}{2} \sum_k \left(|g\rangle\langle k| + |k\rangle\langle g|\right),$$

   where the exciton kinetic energy is neglected assuming $\hbar|\Delta| \gg \hbar^2 k^2/2m_x$.

   We can also assume that $\sum_k 1 = M$, where $M \sim L^3/a_0^3$ is a very large integer (L being the system size and a_0 the exciton Bohr radius).

   a.) [10 pts] Treat the Δ term as H_0, and the Ω term as a perturbation. List all of the bare energy levels, and their degeneracies. Do not forget to include $|g\rangle$.

   b.) [10 pts] Determine the good eigenstates for perturbation theory.

   c.) [10 pts] Use perturbation theory to compute the energy levels of H to first-order in Ω.

   d.) [10 pts] Use perturbation theory to compute the eigenstates of H to first-order in Ω.

   e.) [10 pts]Use perturbation theory to compute the second-order correction to the energy levels.

   f.) [10 pts extra credit] Find the exact eigenvalues and eigenstates of H.