1. [10pts] The trace of an operator is defined as $\text{Tr}\{A\} = \sum_m \langle m | A | m \rangle$, where $\{|m\rangle \}$ is a suitable basis set.
   
   (a) Prove that the trace is independent of the choice of basis.
   (b) Prove the linearity of the trace operation by proving $\text{Tr}\{aA + bB\} = a\text{Tr}\{A\} + b\text{Tr}\{B\}$.
   (c) Prove the cyclic property of the trace by proving $\text{Tr}\{ABC\} = \text{Tr}\{BCA\} = \text{Tr}\{CAB\}$.

2. Consider the system with three physical states $\{|1\rangle, |2\rangle, |3\rangle\}$. In this basis, the Hamiltonian matrix is:

$$H = \begin{pmatrix}
1 & 2i & 1 \\
-2i & 2 & -2i \\
1 & 2i & 1 
\end{pmatrix}$$

Find the eigenvalues $\{\omega_1, \omega_2, \omega_3\}$ and eigenvectors $\{\omega_1, \omega_2, \omega_3\}$ of $H$. Assume that the initial state of the system is $|\psi(0)\rangle = |1\rangle$. Find the three components $\langle 1 | \psi(t) \rangle$, $\langle 2 | \psi(t) \rangle$, and $\langle 3 | \psi(t) \rangle$. Give all of your answers in proper Dirac notation.
