1. Consider a potential consisting of a delta function at $x = 0$ and an infinite barrier at $x = W$,

$$V(x) = \begin{cases} 
g \delta(x); & x < W \\
\infty; & x > W 
\end{cases},$$

(1)

where $g > 0$.

For the case of an incoming wave from the left with of the form $e^{ikx} + re^{-ikx}$, calculate the reflection amplitude $r$, and give an explicit expression for the wavefunction in the region $0 < x < W$. 


2. Consider a standard two-level Rabi system characterized by a detuning $\Delta$ and a ‘bare Rabi frequency’ $\Omega_0$. Let $\{|1\rangle, |2\rangle\}$ be the physical basis, with $|1\rangle$ being the ground state for $\Omega_0 = 0$ and $\Delta > 0$.

The system is initially prepared in state $|1\rangle$, with $\Delta = 0$ and $\Omega_0 = 0$. At $t = 0$, the coupling strength, $\Omega_0$ is suddenly switched to a real, positive value, and then held constant for a duration $T$.

(a) What are the energy eigenvalues and eigenvectors during the interval $0 < t < T$?
(b) What is the state of the system at time $t = T$?
(c) If the observable $D = -i|1\rangle\langle 2| + i|2\rangle\langle 1|$ were measured at time $t = T$, what are the possible results of the measurement?
(d) What are the probabilities of obtaining each possible result?
(e) For each possible result, give the state of the system after the measurement.