PHYS852 Quantum Mechanics II, Fall 2008

HOMEWORK ASSIGNMENT 1:
Density Operator

1. [10pts] The trace of an operator is defined as $Tr\{A\} = \sum_m \langle m|A|m\rangle$, where $\{|m\rangle\}$ is an arbitrary basis set. Introduce a second arbitrary basis set, and use it to prove that the trace is independent of the choice of basis.
2. [10pts] Prove the linearity of the trace operation by proving $Tr\{aA+bB\} = aTr\{A\}+bTr\{B\}$.
3. [10pts] Prove the cyclic property of the trace by proving $Tr\{ABC\} = Tr\{BCA\} = Tr\{CAB\}$.
4. [10pts] Which of the following density matrices correspond to a pure state?

$$\rho_1 = \begin{pmatrix} \frac{2}{7} & 0 \\ 0 & \frac{5}{7} \end{pmatrix} \qquad \rho_2 = \begin{pmatrix} \frac{1}{4} & i\frac{\sqrt{3}}{4} \\ -i\frac{\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix} \qquad \rho_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_4 = \begin{pmatrix} \frac{1}{5} & \frac{\sqrt{2}}{5} \\ \frac{\sqrt{2}}{5} & \frac{4}{5} \end{pmatrix} \qquad \rho_5 = \begin{pmatrix} \frac{1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{2}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix}$$

5. [10 pts] Derive the equation of motion, $\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H,\rho(t)]$, using Schrödinger's equation and the most general form of the density operator, $\rho = \sum_j P_j|\psi_j\rangle\langle\psi_j|$.