PHYS852 Quantum Mechanics II, Spring 2010
HOMEWORK ASSIGNMENT 10

Topics covered: Green's function, Lippman-Schwinger Eq., T-matrix, Born Series.

1. **T-matrix approach to one-dimensional scattering:** In this problem, you will use the Lippman-Schwinger equation
$$|\psi\rangle = |\psi_0\rangle + GV|\psi\rangle, \tag{1}$$
to solve the one-dimensional problem of tunneling through delta potentials. Take $\psi_0(z) = e^{ikz}$, and let
$$V(z) = g\delta(z) + g\delta(z-L). \tag{2}$$
   (a) Express Eq. (1) as an integral equation for $\psi(z)$, and then use the delta-functions to perform the integral. It might be helpful to introduce the dimensionless parameter $\alpha = \frac{Mg}{\hbar^2 k}$. To solve for the two unknown constants, generate two equations by evaluating your solution at $z = 0$, and $z = L$.

   (b) Compute the transmission probability $T = |t|^2$, with t defined via
$$\lim_{z\to\infty} \psi(z) = te^{ikz}. \tag{3}$$
   (c) In the strong-scatterer limit $\alpha \gg 1$, at what k-values is the transmission maximized?

   (d) Consider an infinite square-well of length L. What are the k-values for each bound-state? How do these compare with the transmission resonances in the strong-scatterer limit?

2. **The first Born-approximation:** In the first Born-approximation, find the scattering amplitude, $f(\theta, \phi|k)$, for a Gaussian scattering potential,
$$V(r) = V_0 e^{(-r/r_0)^2}. \tag{4}$$
Still within the first Born-approximation, what is the differential cross-section, $\frac{d\sigma}{d\Omega}$, and total cross-section, σ_{tot}? First try the integral in spherical coordinates, then when you reach the peak of frustration, try switching to Cartesian coordinates.

3. **The Huang-Fermi pseudopotential:** First, try to compute the T-matrix in three dimensions for a three-dimensional delta-function scatter, $V(\vec{r}) = g\delta^3(\vec{r})$. What happens?

   A workable zero-range potential in three-dimensions is called the Huang-Fermi pseudo-potential, V_{HF}, defined via

$$\langle \vec{r} | V_{HF} | \psi \rangle = \delta^3(\vec{r}) \psi_{reg}(\vec{r}), \tag{5}$$

   where

$$\psi_{reg}(\vec{r}) = \frac{d}{dr} r \, \psi(\vec{r}). \tag{6}$$

   This potential is also referred to as a "regularized delta-function".

   (a) By expanding $\psi(\vec{r})$ in powers of r, starting with r^{-1}, show that the effect of the regularization operator, $\frac{d}{dr} r$ is to remove the $1/r$ term in the expansion. Thus $\psi_{reg}(\vec{r})$, is always non-singular at $r = 0$.

   (b) Compute the T-matrix for V_{HF}, using the regularization property to solve the singularity problem encountered with the simple delta-function.

   (c) Use your answer to part (b) to compute the differential cross-section, $\frac{d\sigma}{d\Omega}$, as well as the total cross-section, σ_{tot}, for the Huang-Fermi pseudo-potential.