PHYS852 Quantum Mechanics II, Spring 2010
HOMEWORK ASSIGNMENT 5

Topics covered: rotation with spin, exchange symmmetry

1. The Hamiltonian for the deuteron, a bound-state of a proton and neutron, may be written in the form
$$H = \frac{P_p^2}{2M_p} + \frac{P_n^2}{2M_n} + V_1(R) + V_2(R)\vec{S}_p \cdot \vec{S}_n, \tag{1}$$
where R is the relative radial coordinate. Both are spin-1/2 particles, but they are not identical.
   (a) The total angular momentum operator is $\vec{S} = \vec{S}_p + \vec{S}_n$. The state $|s_p s_n s\, m\rangle$ is the simultaneous eigenstate of $\vec{S}_p$, $\vec{S}_n$, S^2, and S_z. What are the allowed values of the total spin quantum number s? For each s-value, what are the allowed m quantum numbers.
   (b) Show that $|s_p s_n s\, m\rangle$ is an eigenstate of $\vec{S}_p \cdot \vec{S}_n$, and give the corresponding eigenvalue. Hint, use the fact that $S^2 = (\vec{S}_p + \vec{S}_n) \cdot (\vec{S}_p + \vec{S}_n)$.
   (c) Give ten distinct quantum numbers that can be assigned to an eigenstates of this H. Note that this includes s_p and s_n, even though they can never change.
   (d) What one-dimensional wave equation would you have to solve to find the energy eigenvalue associated with one of these states?

2. Consider a particle of spin $s = 1$, constrained to move on the surface of a sphere. Assume that the Hamiltonian of the particle is
$$H = \frac{L^2}{2I} + \frac{\vec{L} \cdot \vec{S}}{I}, \tag{2}$$
where $\vec{L}$ is the orbital angular momentum operator, I is the moment of inertia, and $\vec{S}$ is the spin operator. find the quantized energy levels and the degeneracy of each level.

3. For Silicon, the ground-state configuration is $(3p)^2$, i.e. there are two valence electrons, each in the $3p$ state.
   (a) What are the possible values for the total spin quantum number, s, where $\vec{S} = \vec{S}_1 + \vec{S}_2$?
   (b) What are the possible values for the total angular momentum quantum number, ℓ, where $\vec{L} = \vec{L}_1 + \vec{L}_2$?
   (c) The exchange symmetry of the two-electron spatial wavefunction matches the parity of the quantum number ℓ. Based on this, determine which combinations of s and ℓ are allowed states for the two-electron system.
   (d) For each allowed combination, what are the possible values of the quantum number j, where $\vec{J} = \vec{L} + \vec{S}$?
   (e) Assuming that the spin-orbit interaction lifts the degeneracy of the states with different j, how many distinct energy levels make up the fine-structure of the $(3p)^2$ state?
   (f) Which j levels would shift if a contact interaction between the two valence electrons were added to the Hamiltonian?

4. Let $\vec{J} = \vec{L} + \vec{S}$. Using the method described in the lecture, identify and calculate all non-zero Clebsch-Gordan coefficients for the $\ell = 2$, $s = 1/2$ case.