PHYS852 Quantum Mechanics II, Spring 2010
HOMEWORK ASSIGNMENT 6

Topics covered: Time-independent perturbation theory.

1. [30] **Two-Level System**: Consider the system described by $H = \delta S_z + \Omega S_x$, with $\delta > 0$, where S_x and S_z are components of the spin vector of an $s = 1/2$ particle. Treat the S_z term as the bare Hamiltonian.

   (a) [15] Use perturbation theory to compute the eigenvalues and eigenvectors of H. Compute all terms up to fourth-order in Ω.

   (b) [5] Expand the exact eigenvalues and eigenvectors around $\Omega = 0$ and compare to the perturbation theory results.

   (c) [10] Verify that the states computed in (a) are normalized to unity and orthogonal up to fourth-order.

2. [20] **Resonance-frequency shifts**: Consider a system with a 3-dimensional Hilbert space spanned by states $|a\rangle$, $|b\rangle$, and $|c\rangle$. In the basis $\{|a\rangle, |b\rangle, |c\rangle\}$, let the bare Hamiltonian of the system be

$$H_0 = \hbar\Delta \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 3 \end{pmatrix}. \tag{1}$$

   For the case where the system is perturbed by the operator

$$V = \hbar\chi \begin{pmatrix} 1 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \tag{2}$$

   also given in $\{|a\rangle, |b\rangle, |c\rangle\}$ basis. Calculate the *shifts* in the *resonance frequencies* of the full system relative to those of the unperturbed system, to second-order in χ.

3. [15] Consider a pair of quantum harmonic oscillators, described by the bare Hamiltonian

$$H_0 = \hbar\omega(A^\dagger A + 1/2) + \hbar\Omega(B^\dagger B + 1/2).$$

   Assume that $\omega < \Omega < 2\omega$, and determine the three lowest bare energy eigenvalues and eigenvectors. Consider the perturbation

$$V = \hbar g \left(A^\dagger A^\dagger B + B^\dagger A A \right).$$

   Show that two of the three lowest levels are exact eigenstates of $H = H_0 + V$. For the remaining bare level, compute the first non-vanishing corrections to the eigenvalue and eigenvector.

4. [15] Consider a particle of mass M confined to a 1-dimensional box of length L. Use perturbation theory to caculate the effects of adding a tilt to the box, represented by adding the linear potenital

$$V_{tilt}(x) = \hbar\beta \left(\frac{x}{L} - \frac{1}{2} \right)$$

   to the box potential,

$$V_{box}(x) = \begin{cases} 0; & 0 < x < L \\ \infty; & \text{else} \end{cases}$$

   Calculate the three lowest perturbed eigenstates to first-order and their corresponding eigenvalues to second-order.