PHYS852 Quantum Mechanics II, Spring 2010
HOMEWORK ASSIGNMENT 7

Topics covered: addition of three angular momenta, degenerate perturbation theory

1. Consider a system of two spin-1/2 particles, described by $\vec{S}_1$ and $\vec{S}_2$, and one spin-1 particle, described by $\vec{S}_3$. Let $\vec{S}' = \vec{S}_1 + \vec{S}_2$. What are the allowed values of s'? For each allowed value, give the allowed m' values. Use Clebsch Gordan coefficients to express the $|s'm'm_3\rangle$ states in terms of the $|m_1m_2m_3\rangle$ states.

   Let $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$. What are the allowed values of the quantum number s? For each s value, list the alllowed m-values. Express the states $|s', s, m\rangle$ as linear superpositions of the $|s'm'm_3\rangle$ states, and then as linear superpositions of the $|m_1m_2m_3\rangle$ states.

2. Derive general expressions for the energy-shifts up to third-order and state vectors up to second-order in degenerate perturbation theory.

3. Consider a system described in the basis $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ by the bare Hamiltonian

$$H_0 = \hbar\omega_0 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \tag{1}$$

   Let the system be perturbed by the operator

$$V = \hbar g \begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix} \tag{2}$$

   First, determine the 'good basis' for degenerate perturbation theory, then compute the eigenvalues and eigenvectors of $H = H_0 + V$ to second-order in perturbation theory.

4. Prove that in order to predict the resonance-frequency shifts of a perturbaed quantum system to second-order, one needs only to compute the energy levels to second-order, and that this implicitly requires computing the eigenstates only to first-order.

   Then show that in order to compute the shift in the expectation-value of an arbitrary operator to second-order, one must compute the eigenstates to second-order.

5. Consider a pair of identical harmonic oscillators, described by the Hamiltonian

$$H_0 = \hbar\omega(A_1^\dagger A_1 + A_2^\dagger A_2 + 1). \tag{3}$$

   Determine the energy eigenvalues and degeneracies of the two lowest energy levels of H_0.

   Let the system be perturbed by the operator

$$V = \hbar\lambda(A_1^\dagger A_2^\dagger + A_1^\dagger A_2 + A_2^\dagger A_1 + A_1 A_2). \tag{4}$$

   For the states comprising the two lowest bare energy levels, determine the full energy eigenvalues of $H = H_0 + V$ to second-order and the eigenstates to first-order in λ.