# Optical traps for electron produced by Pauli blocking

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**Abstract.** - We propose a mechanism to trap electrons in a semiconductor using counter-propagating laser beams. The trapping comes from Pauli blocking between electron and virtual exciton coupled to unabsorbed photons. Electron exchange allows for the possibility of momentum transfer between photon and electron. This leads to a sinusoidal trap for electron with a period determined by the laser beam modulation. The depth of the trap is proportional to the laser intensity and inversely proportional to the exciton-photon detuning.

**Introduction.** – One of the most interesting aspects of atomic optical traps [1, 2] is the possibility of a direct probing of many-body physics in a framework in which the particle position can be precisely controlled [3]. Optical traps have been used to explore the rich physics of fermionic and bosonic gases of cold atoms, in particular the superfluid to Mott insulator phase transition [4]. In these systems, particle trapping occurs in regular lattices, and is obtained by interfering counter-propagating laser beams. Recently, a similar idea for trapping electrons in a semiconductor system has been proposed [5]. In that scheme, the electron trapping takes advantage of the trion resonance related to the existence of a bound state for two electrons (in a singlet state) and one hole [6]. Even if the electron is difficult to trap because its mass is much lighter than the atomic mass, the large optical dipole moment of the semiconductor trionic transition can make up for the particle kinetic energy by producing a deep trapping potential. Indeed, it was shown in Ref. [5] that the ratio of the trapping potential depth over the particle characteristic kinetic energy, i.e., the recoil energy following the scattering by a photon, is comparable in atomic and semiconductor systems.

In this letter, we focus on a different configuration for the electron spin and laser polarization that does not use the bound trion state as a virtual intermediate state in the trapping. The spin configuration considered here - which leads to a triplet instead of a singlet state for the two electrons - produces a trapping only due to the Pauli exclusion principle, in the absence of Coulomb process. The effect can be understood as follows: In a quantum well, a laser beam with $\sigma_+$ circular polarization tends to add virtual excitons made of $(-1/2)$ electrons. This possibility is blocked at a particular position of the sample if an electron with same (-1/2) spin is already there. This leads to a spatial modulation of the electron that can be described through an effective trapping potential. A similar mechanism for trapping a photo-excited electron-hole plasma has been discussed by Lindberg and Binder [8, 9]. More recently, experimental methods for realizing traps for excitons and polaritons in semiconductors have followed different strategies based on local strain field [10], local metallic gates [11], repulsive exciton-exciton [12] and polariton-polariton [13, 14] interaction, and even acoustic waves [16] following an original proposal by Ivanov and Littlewood [15]. The present scheme focuses on a different situation, the semiconductor being here doped with electrons at very low density. Unlike electron-hole plasma, excitons or polaritons, these electrons do not recombine but remain in the system for a time long enough to allow for applications similar to the ones of atomic systems. The optical traps for electrons we here discuss, will produce "dynamic quantum dots", similar to the ones realized in semiconductors using surface acoustic waves [17, 18].

The usual approach to optical trapping in atomic physics relies on the adiabatic theorem [19] through an effective Hamiltonian in which the light-matter coupling enters via second order perturbation theory. Here, we propose a different formulation that directly calculates the time evolution of an electron with a given momentum $\mathbf{k}$

after turning on two unabsorbed counter-propagating laser beams. The explicit form of this time evolution is calculated to lowest (second) order in the light-matter coupling. Using it, we can derive the effective trapping potential felt by the electron. This approach is a priori equivalent to the standard one. It however provides a neat view of the physical requirements to possibly define such a potential. Moreover, the theoretical formulation proposed here will allow an easy extension to trapping of more complex structures than single electrons.

This letter is organized as follows: We first explain the physical idea behind the mixing of electron plane waves induced by interaction with two counter-propagating laser beams. We then provide a detailed calculation of the electron time evolution after the laser field is turned on. We derive a closed analytical expression of the transition rate leading to the electron trapping. This section also discusses the two regimes corresponding to times much shorter and much longer than the inverse of the photon detuning. Finally, we show how to derive an effective trapping potential from the scattering rate of the electron wave mixing.

**Physical idea.** – Excitons are single electron-hole pairs, eigenstate of the semiconductor Hamiltonian. Let us denote as $B_I^\dagger$ their creation operators. The index $I$ has an orbital part $i = (\nu_i, \mathbf{K}_i)$ for exciton with center of mass momentum $\mathbf{K}_i$ and relative motion index $\nu_i$ which includes both, discrete and continuum states. The index $I$ also has a spin part $(s_i, m_i)$ for the electron and the hole making the exciton. $s_i = (\pm 1/2)$ for conduction electrons while $m_i = (\pm 3/2, \pm 1/2)$ for holes in bulk samples. For narrow quantum wells however, we can restrict to heavy holes with $m_i = (\pm 3/2)$, light holes being far below in energy.

Coupling between semiconductor and electromagnetic field with frequency $\omega_P$ can be handled in the rotating wave approximation, which leads to a time-independent formulation of the problem, the energy of the conduction-band electron energy then reading as

$$\epsilon^e_{\mathbf{k}} = E_{gap} + \frac{\mathbf{k}^2}{2m_e} - \omega_P, \tag{1}$$

where $E_{gap}$ is the band-gap energy, and $m_e$ the electron effective mass. Since the light-matter coupling conserves momentum and spin, coupling to a standing-wave made of circular polarized photons $\sigma$ with momenta $(\mathbf{Q}, -\mathbf{Q})$ reads as $W_\sigma = U^\dagger_\sigma + U_\sigma$, with

$$U^\dagger_\sigma = \frac{1}{\sqrt{2}} \left( U^\dagger_{\mathbf{Q}\sigma} + U^\dagger_{-\mathbf{Q}\sigma} \right) , \tag{2}$$

where $U^\dagger_{\mathbf{Q}\sigma} = \sum_\nu \Omega_\nu B^\dagger_{\nu\mathbf{Q}\sigma}$, creates a semiconductor excitation similar to the laser field mode. The Rabi energy, $\Omega_\nu$, is related to sample size, $L$, and dimensionality, $D$, via $\Omega_\nu = \Omega L^{D/2} \langle \nu | \mathbf{r} = 0 \rangle$, where $\Omega$, proportional to the laser amplitude, is the energy of the interband transition while $\langle \nu | \mathbf{r} \rangle$ is the exciton relative motion wavefunction. In quantum well, $\sigma_\pm$ excitons are made of $\mp 1/2$ electrons and $\pm 3/2$ holes. In bulk, they are also made of $\pm 1/2$ electrons and $\pm 1/2$ light holes. However, since photon coupling to light-hole excitons is three times weaker than coupling to heavy-hole excitons, we will focus here on the heavy-hole contributions.

Let us consider a semiconductor having one excess conduction-band electron with momentum $\mathbf{k}$ and spin $s$. At time $t = 0$, we irradiate the sample with a standing-wave laser field having a circular polarization $\sigma_+$, momenta $(+\mathbf{Q}, -\mathbf{Q})$, and frequency $\omega_P$ tuned far away from the exciton resonances. This makes the material nearly transparent, so that we can ignore photon absorption. Off-resonant photons interact with the material via the creation and annihilation of delocalized virtual excitons. The Lorentz force on the conduction electron results in a ponderomotive potential [21], too weak to be of interest. An indirect coupling between the electron and the laser field also exists due to interactions between the conduction electron and the virtual excitons coupled to the unabsorbed photons via both, the Coulomb potential and the Pauli exclusion principle. When the electron has a spin $1/2$, it interacts with the $\sigma_+$ virtual excitons via Coulomb forces only. When the electron has a spin $-1/2$, it also feels the electron of the virtual excitons through the Pauli exclusion principle. This appears via the possibility of an electron exchange between the real electron and the virtual exciton [23]. The fact that the scattering associated to electron exchange is dimensionless, makes the interaction of an electron with an unabsorbed laser beam dominantly controlled by this electron exchange in the absence of Coulomb process: Indeed, due to a simple dimensional argument, scatterings in which Coulomb interaction enters, have to appear with an energy denominator which can only be a photon detuning, this detuning being large in the case of unabsorbed photons.

Figure 1 shows the two distinct exchange channels with excitons formed out of a standing wave made of $(\mathbf{Q},-\mathbf{Q})$ photons. While in Fig.1(a), the electron momentum remains unchanged, in Fig.1(b), it changes from $\mathbf{k}$ to either $\mathbf{k} + 2\mathbf{Q}$ or $\mathbf{k} - 2\mathbf{Q}$. As a result, the electron distribution is modulated on a scale $1/Q$ with a rate proportionnal to the laser intensity. Indeed, according to the exchange processes shown in Fig.1, an electron with momentum $\mathbf{k}$ and spin $s$, must evolves in an unabsorbed standing wave with momenta $(\mathbf{Q}, -\mathbf{Q})$ and circular polarization $\sigma$, according to

$$|\mathbf{k}, s\rangle_t = |\mathbf{k}, s\rangle + \sum_{\mathbf{q}=0,\pm 2\mathbf{Q}} \gamma_t(\mathbf{k}s, \mathbf{q}; \sigma) |\mathbf{k} + \mathbf{q}, s\rangle \ , \tag{3}$$

the prefactor $\gamma_t(\mathbf{k}s, \mathbf{q}; \sigma)$ being proportional to $\Omega^2$ and different from zero for $s = -\sigma/2$ only. Eq. 3 corresponds to a sinusoidal modulation of the electron density as easy to see from the electron probability distribution calculated

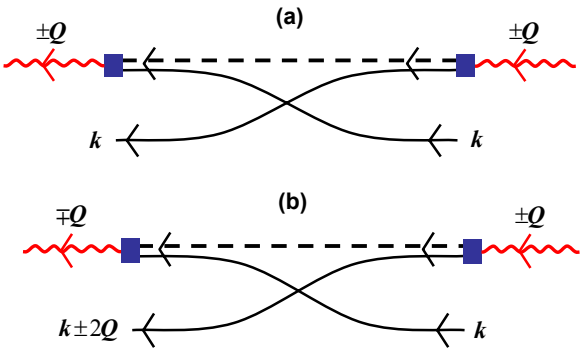


Fig. 1: Exchange-assisted electron-photon scattering in the presence of a standing wave. (a) Diagonal terms. (b) Off-diagonal terms giving rise to the optical trap. Electrons are represented by solid lines, holes by dashed lines.

through

$$\frac{|\langle \mathbf{r}|\mathbf{k},s\rangle_t|^2}{{}_t\langle \mathbf{k},s|\mathbf{k},s\rangle_t} \simeq \frac{1}{L^D}\left\{1+2|A|\cos\left(2\mathbf{Q}\cdot\mathbf{r}+\varphi\right)\right\} \quad (4)$$

where we have set

$$\gamma_t(\mathbf{k}s,2\mathbf{Q};\sigma)+\gamma_t^*(\mathbf{k}s,-2\mathbf{Q};\sigma)=|A|e^{i\varphi}\ . \quad (5)$$

Two counter-propagating photon beams with $\sigma_+$ circular polarization thus tend to localize $s=-1/2$ electrons on planes in 3D and stripes in 2D, with a periodicity $\pi/Q$ where $Q$ is the photon momentum in 3D and its projection over the quantum well plane in 2D.

**Time evolution of electron states.** – The dynamics of the electron modulation can be calculated by looking at the time evolution of the electron state $|\mathbf{k},s\rangle$ when a laser beam is turned on at $t=0$. Since the coupling $W$ to unabsorbed photons creates or destroys one virtual exciton, the resulting state $e^{-i(H_{SC}+W)t}|\mathbf{k},s\rangle$ where $H_{SC}$ is the semiconductor Hamiltonian in the rotating frame, is made of a superposition of states having one real electron plus $n$ virtual excitons. The projection of this state on the one-electron subspace can be written, within an irrelevant phase factor, as

$$|\mathbf{k},s\rangle_t=|\mathbf{k},s\rangle+\sum_{\mathbf{k}',s'}c_{\mathbf{k}'s',\mathbf{k}s}(t)|\mathbf{k}',s'\rangle\ , \quad (6)$$

where

$$c_{\mathbf{k}'s',\mathbf{k},s}(t)=\langle \mathbf{k}'s'|e^{-i(H_{SC}+W-\epsilon^e_{\mathbf{k}})t}-1|\mathbf{k},s\rangle. \quad (7)$$

In order to calculate $c_{\mathbf{k}'s',\mathbf{k}s}(t)$, we use the integral representation of the exponential and we expand $(a-H_{SC}-W)^{-1}$ in terms of $W$. The zero and first order terms cancel while the $W^2$ term gives, since $U|\mathbf{k},s\rangle=0$,

$$c_{\mathbf{k}'s',\mathbf{k}s}(t)\simeq\int_{-\infty}^{\infty}\frac{dx}{-2i\pi}e^{-i(x+i0_+-\epsilon^e_{\mathbf{k}})t}\langle\mathbf{k}',s'|\frac{1}{x+i0_+-H_{SC}}$$
$$\times U\frac{1}{x+i0_+-H_{SC}}U^\dagger\frac{1}{x+i0_+-H_{SC}}|\mathbf{k},s\rangle\ , \quad (8)$$

where $0_+$ is an arbitrary positive constant. $H_{SC}$ acting on the right gives the free electron $\mathbf{k}$ energy $\epsilon^e_{\mathbf{k}}$ while on the left, it gives $\epsilon^e_{\mathbf{k}'}$. Integration using the residue theorem then gives

$$c_{\mathbf{k}'s',\mathbf{k}s}(t)=\langle\mathbf{k}',s'|UG_t\left(\epsilon^e_{\mathbf{k}'}-\epsilon^e_{\mathbf{k}},H_{SC}-\epsilon^e_{\mathbf{k}}\right)U^\dagger|\mathbf{k},s\rangle \quad (9)$$

where $G_t(E,E')=G_t(E',E)$, defined as

$$G_t(E,E')=\frac{1}{EE'}+\frac{e^{-iEt}}{E(E-E')}+\frac{e^{-iE't}}{E'(E'-E)}\ .$$
$$=\frac{F_t(E)-F_t(E')}{E-E'} \quad (10)$$

is the standard function for second order time evolution [22] while $F_t(E)$, defined as

$$F_t(E)=\frac{e^{-iEt}-1}{E}=-2i\pi e^{-iEt/2}\delta_t(E) \quad (11)$$

is the characteristic function at first order, $\delta_t(E)=(\pi E)^{-1}\sin Et/2$ ensuring energy conservation at the $1/t$ scale. The function $F_t(E)$ tends to $-it$ for $|Et|$ small and to zero for $|Et|$ large.

Since $H_{SC}$ conserves spin and momentum, the matrix element in Eq. 9 differs from zero for $s'=s$ and $\mathbf{k}'+\eta'\mathbf{Q}=\mathbf{k}+\eta\mathbf{Q}$ with $\eta,\eta'=\pm1$ only. This leads to $\mathbf{k}'=\mathbf{k}+\mathbf{q}$ with $\mathbf{q}=(0,\pm2\mathbf{Q})$. The diagonal term $\mathbf{k}=\mathbf{k}'$ comes from $(U,U^\dagger)$ replaced by $(U_{\mathbf{Q}},U^\dagger_{\mathbf{Q}})$ or $(U_{-\mathbf{Q}},U^\dagger_{-\mathbf{Q}})$, while the term $\mathbf{k}'=\mathbf{k}+2\eta\mathbf{Q}$ comes from $(U,U^\dagger)$ replaced by $(U_{-\eta\mathbf{Q}},U^\dagger_{\eta\mathbf{Q}})$. This shows that for standing wave having a circular polarization $\sigma$, Eq.6 reduces to Eq.3 for

$$\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)=\frac{1}{2}\langle\mathbf{k}+2\eta\mathbf{Q},s|U_{-\eta\mathbf{Q}\sigma}$$
$$\times G_t\left(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}},H_{SC}-\epsilon^e_{\mathbf{k}}\right)U^\dagger_{\eta\mathbf{Q}\sigma}|\mathbf{k},s\rangle. \quad (12)$$

Since $F_{t=0}(E)=0$, the above quantity cancels for $t=0$ as expected. To calculate it for finite $t$, we first note that, in Eq. 12, $H_{SC}$ acts on two electrons plus one hole. The exact calculation of $\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)$ thus requires the knowledge of the *whole* trion eigenstate spectrum. In its absence, it is yet possible to calculate it through an expansion in the Coulomb interaction between electron and exciton, this interaction being small since excitons are neutral. Due to dimensional arguments, this expansion actually corresponds to an expansion in the inverse of the photon detuning. The operator $H_{SC}$ acting on an electron-exciton pair then gives, to leading order in the inverse detuning,

the sum of the free electron and free exciton energies; so that

$$G_t(\epsilon^e_{\mathbf{k}'}-\epsilon^e_{\mathbf{k}}, H_{SC}-\epsilon^e_{\mathbf{k}})U^\dagger_{\eta\mathbf{Q}\sigma}|\mathbf{k},s\rangle \simeq \sum_\nu \Omega_\nu G_t(\epsilon^e_{\mathbf{k}'}-\epsilon^e_{\mathbf{k}}, E^X_{\nu\mathbf{Q}})B^\dagger_{\nu,\eta\mathbf{Q}\sigma}|\mathbf{k},s\rangle \quad (13)$$

where the exciton energy in the rotating frame reads as

$$E^X_{\nu\mathbf{Q}} = E_{Gap}+\epsilon_\nu + Q^2/2(m_e+m_h)-\omega_P. \quad (14)$$

This allows us to rewrite Eq.12 as

$$\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma) = \frac{1}{2}\sum_\nu \Omega_\nu \Lambda_{\nu\sigma}(\mathbf{k}s,2\eta\mathbf{Q})\frac{F_t(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})-F_t(E^X_{\nu\mathbf{Q}})}{(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})-E^X_{\nu\mathbf{Q}}} \quad (15)$$

where the electron-photon momentum transfer is controlled by

$$\Lambda_{\nu\sigma}(\mathbf{k}s,2\eta\mathbf{Q}) = \langle \mathbf{k}+2\eta\mathbf{Q},s|U_{-\eta\mathbf{Q}\sigma}B^\dagger_{\nu,\eta\mathbf{Q}\sigma}|\mathbf{k},s\rangle \ . \quad (16)$$

This transfer physically comes from electron exchange between the real electron $(\mathbf{k},s)$ and the virtual exciton $(\nu,\eta\mathbf{Q},\sigma)$, as can be seen by using our recently developed many-body formalism for composite boson excitons [23] extended to electron-exciton systems.

In this formalism, carrier exchange follows from two commutators

$$\left[B_{j'\sigma'},B^\dagger_{j\sigma}\right] = \delta_{j'j}\delta_{\sigma'\sigma}-D_{j'\sigma',j\sigma}$$
$$\left[D_{j'\sigma',j\sigma},a^\dagger_{\mathbf{k}s}\right] = \sum_{\mathbf{k}'s'}\lambda\begin{pmatrix}j'\sigma' & j\sigma\\ \mathbf{k}'s' & \mathbf{k}s\end{pmatrix}a^\dagger_{\mathbf{k}'s'} \quad (17)$$

The dimensionless Pauli scattering $\lambda\begin{pmatrix}j'\sigma' & j\sigma\\ \mathbf{k}'s' & \mathbf{k}s\end{pmatrix}$ between one exciton and one electron in the absence of Coulomb process, is readily obtained from the diagrams of Fig.1: The spin part imposes $\sigma'=\sigma$ and $s=s'=-\sigma/2$. To get the orbital part $\lambda\begin{pmatrix}j' & j\\ \mathbf{k}' & \mathbf{k}\end{pmatrix}$, we write the wave functions of the "in" states $(\mathbf{k},j)$ and the complex conjugates of the wave functions of the "out" states $(\mathbf{k}',j')$, and we integrate over all carrier positions [23]. This leads to

$$\lambda\begin{pmatrix}j' & j\\ \mathbf{k}' & \mathbf{k}\end{pmatrix} = \int d\mathbf{r}_e d\mathbf{r}'_e, d\mathbf{r}_h \times\langle\nu_{j'}|\mathbf{r}'_e-\mathbf{r}_h\rangle\langle\mathbf{Q}_{j'}|\alpha_e\mathbf{r}'_e+\alpha_h\mathbf{r}_h\rangle\langle\mathbf{k}'|\mathbf{r}_e\rangle \times\langle\mathbf{r}_e-\mathbf{r}_h|\nu_j\rangle\langle\alpha_e\mathbf{r}_e+\alpha_h\mathbf{r}_h|\mathbf{Q}_j\rangle\langle\mathbf{r}'_e|\mathbf{k}\rangle \quad (18)$$

where $\alpha_e=1-\alpha_h=m_e/(m_e+m_h)$. By writing $\langle\mathbf{r}|\nu\rangle$ as $\sum_{\mathbf{p}}\langle\mathbf{r}|\mathbf{p}\rangle\langle\mathbf{p}|\nu\rangle$, it can be shown that this Pauli scattering reduces to

$$\lambda\begin{pmatrix}j' & j\\ \mathbf{k}' & \mathbf{k}\end{pmatrix} = \delta_{\mathbf{k}'+\mathbf{Q}',\mathbf{k}+\mathbf{Q}}\langle\nu_{j'}|\mathbf{k}-\alpha_e\mathbf{Q}_{j'}\rangle\langle\mathbf{k}'-\alpha_e\mathbf{Q}_j|\nu_j\rangle \ . \quad (19)$$

This formalism allows us to rewrite Eq.16 as

$$\Lambda_{\nu\sigma}(\mathbf{k}s,2\eta\mathbf{Q}) = -\delta_{s,-\sigma/2}\sum_{\nu'}\Omega^*_{\nu'}\langle\nu'|\mathbf{k}+\alpha_e\eta\mathbf{Q}\rangle\langle\mathbf{k}+(1+\alpha_h)\eta\mathbf{Q}|\nu\rangle \quad (20)$$

which, after summing over $\nu'$ through closure relation, further simplifies into

$$\Lambda_{\nu\sigma}(\mathbf{k}s,2\eta\mathbf{Q}) = -\delta_{s,-\sigma/2}\Omega^*\langle\mathbf{k}+(1+\alpha_h)\eta\mathbf{Q}|\nu\rangle \ . \quad (21)$$

Let us now turn to the time dependence of $\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)$ given in Eq.15. We first note that, since the photon momentum $\mathbf{Q}$ is very small on the electronic scale, the relevant exciton energies $E^X_{\nu\mathbf{Q}}$, of the order of the photon detuning, are large compared to $(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})$. Three time regimes then have to be considered:

(i) The first one corresponds to $t$ small enough to have $(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})t \ll E^X_{\nu\mathbf{Q}}t \ll 1$. The ratio in Eq.15 then tends to $-t^2/2$; so that, using $\Omega_\nu=\Omega L^{D/2}\langle\nu|r=0\rangle$, we get

$$\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)\simeq \delta_{s,-\sigma/2}\frac{t^2|\Omega|^2}{4}\sum_\nu L^{D/2}\langle\mathbf{k}+(1+\alpha_h)\eta\mathbf{Q}|\nu\rangle\langle\nu|r=0\rangle \quad (22)$$

the sum over $\nu$, performed through closure relation, reducing to 1 since $\langle\mathbf{p}|\mathbf{r}=\mathbf{0}\rangle = L^{-D/2}$.

(ii) Under increasing $t$, we reach times such that $(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})t \ll 1 \ll E^X_{\nu\mathbf{Q}}t$. The second $F_t$ in Eq. 15 then goes to zero, while the first $F_t$ tends to $-it$; so that

$$\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)\simeq -it\delta_{s,-\sigma/2}\mathcal{V}(\mathbf{k},2\eta\mathbf{Q}) \quad (23)$$

where the modulation rate is given by

$$\mathcal{V}(\mathbf{k},2\eta\mathbf{Q}) = \frac{|\Omega|^2}{2}\sum_\nu L^{D/2}\frac{\langle\mathbf{k}+(1+\alpha_h)\eta\mathbf{Q}|\nu\rangle\langle\nu|r=0\rangle}{E^X_{\nu\mathbf{Q}}} \ . \quad (24)$$

Since the exciton ground state $\nu_0$ is rather far from the other exciton levels, the $E^X_{\nu\mathbf{Q}}$ denominator for photons tuned close to the exciton ground state level, makes the above sum controlled by $\nu=\nu_0$. To calculate this modulation rate, we then use, for $a_X$ being the $3D$ Bohr radius, $\langle\nu_0|\mathbf{r}=\mathbf{0}\rangle = x_D/a_X^{D/2}$ with $x_D=1/\sqrt{\pi}$ in 3D and $\sqrt{8/\pi}$ in 2D while $\langle\mathbf{k}=\mathbf{0}|\nu_0\rangle = (a_X/L)^{D/2}y_D$ with $y_D=8\sqrt{\pi}$ in 3D and $\sqrt{2\pi}$ in 2D. By noting that the photon momenta is much smaller than the characteristic electron momenta, we end with

$$\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)\simeq -it\delta_{s,-\sigma/2}\frac{|\Omega|^2}{E^X_{\nu,\mathbf{Q}=0}}\frac{\langle\mathbf{k}|\nu_0\rangle}{\langle\mathbf{k}=\mathbf{0}|\nu_0\rangle}\zeta_D \quad (25)$$

where $\zeta_D=x_Dy_D/2$ is equal to 4 in 3D and 2 in 2D.

(iii) The third regime, with $t$ large enough to have $1 \ll (\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}}-\epsilon^e_{\mathbf{k}})t$, is physically irrelevant because it would correspond to times larger than the coherence time, difference in electron energies scaling as the coherence time inverse.

The above discussion shows that $\gamma_t(\mathbf{k}s,2\eta\mathbf{Q};\sigma)$ starts by rising as $\sim|\Omega|^2t^2$, while for $t$ larger that the inverse of the optical detuning, it saturates to $\sim|\Omega|^2t/E^X_{\nu_0\mathbf{Q}=\mathbf{0}}$, leading to a space dependent transition rate controlled by $\sim|\Omega|^2/E^X_{\nu_0\mathbf{Q}=\mathbf{0}}$: This just is what we expect from simple

dimensional arguments, for an energy-like quantity with quadratic coupling to unabsorbed photons.

We wish to mention that the time evolution of the $|\mathbf{k}s\rangle$ state also has contributions on states with one electron plus one or more virtual electron-hole pairs. Contributions from two electrons plus one hole even are of the first order in the light-matter coupling $W$. However, a similar calculation shows that the time dependence of this contribution is in $F_t(E^X_{\nu,\mathbf{Q}})$; so that it tends to zero for time larger than the inverse detuning.

A standing wave $(\mathbf{Q}, -\mathbf{Q})$ with circular polarization $\sigma = +1$ thus tends to trap $s = -1/2$ electrons, according to Eq.3 for times larger than the inverse of the exciton-photon detuning. To equally trap $s = 1/2$ electrons, we can use linear polarization. Indeed, the operator $U$ in Eq.8 then reads as $(1/2)\sum_{\eta=\pm1,\sigma=\pm1} U_{\eta\mathbf{Q},\sigma}$. Due to spin conservation for hole, matrix elements with $U_{\eta'\mathbf{Q},\sigma'}U^\dagger_{\eta\mathbf{Q},\sigma}$ differ from zero for $\sigma = \sigma'$ only. Calculation of the modulation rate then follows from the one done for circular polarization. This readily shows that the $\sigma = +1$ part of the light traps the $-1/2$ component of the electron spin while the $\sigma = -1$ part equally traps the $s = 1/2$ component.

**Effective trapping potential.** – It is possible to describe the time evolution of the electron state $|\mathbf{k}, s\rangle$ given in Eqs.(3,25) through an effective trapping potential $V_{eff}$. Indeed, the integral representation of the exponential gives, to the lowest order in this potential,

$$\begin{aligned} |\mathbf{k}, s\rangle_t &= e^{-i(H_{SC}+V_{eff}-\epsilon^e_{\mathbf{k}})t}|\mathbf{k}, s\rangle \\ &\simeq |\mathbf{k}s\rangle + F_t(H_{SC} - \epsilon^e_{\mathbf{k}})V_{eff}|\mathbf{k}, s\rangle \end{aligned} \quad (26)$$

with $F_t(E)$ defined in Eq.(11). It is then easy to check that the above equation reproduces the relevant time regime $(\epsilon^e_{\mathbf{k}+2\eta\mathbf{Q}} - \epsilon^e_{\mathbf{k}})t \ll 1 \ll E^X_{\nu\mathbf{Q}}t$ associated to Eq.25, provided that the effective trapping potential reads for $\mathbf{q} = \pm 2\mathbf{Q}$ transitions as

$$\begin{aligned} V_{eff} &\simeq \zeta_D \frac{|\Omega|^2}{E^X_{\nu_0,\mathbf{0}}} \\ &\times \sum_{\mathbf{p},\eta=\pm1} \frac{\langle \mathbf{p}|\nu_0\rangle}{\langle \mathbf{p}=\mathbf{0}|\nu_0\rangle} |\mathbf{p}+\eta\mathbf{Q}, -\sigma/2\rangle\langle \mathbf{p}-\eta\mathbf{Q}, -\sigma/2| \ , \end{aligned} \quad (27)$$

the $Q$ dependence in the prefactor being possibly neglected as consistent with $Qa_X \ll 1$. This effective potential tends to spatially trap the electron as readily seen by writing it in real space using $|\mathbf{p}\rangle = \int d\mathbf{r}|\mathbf{r}\rangle\langle\mathbf{r}|\mathbf{p}\rangle$. We then find the following non-local potential

$$\begin{aligned} V_{eff} = \frac{|\Omega|^2}{E^X_{\nu_0,\mathbf{0}}} \int d\mathbf{r}' d\mathbf{r} |\mathbf{r}', -\sigma/2\rangle\langle \mathbf{r}, -\sigma/2| \\ \times K_D(\mathbf{r}' - \mathbf{r}) \cos[\mathbf{Q}\cdot(\mathbf{r}' + \mathbf{r})] \ , \end{aligned} \quad (28)$$

where $K_D(\mathbf{r}) = \langle \mathbf{r}|\nu_0\rangle\langle\nu_0|\mathbf{r} = \mathbf{0}\rangle$ reduces for 2D and 3D to $(x_D^2/a_X^D)e^{-(4-D)r}$. We note that the prefactor in $V_{eff}$ is positive for photon energy below the exciton resonance but turns negative above resonance. In the case of a sinusoidal trap, the potential always has negative regions so that the electron is trapped whatever the sign of the $V_{eff}$ prefactor. By contrast, this sign is important if we want to construct a single trap by using waves with properly adjusted $\mathbf{Q}$ distributions: photons will then have to be tuned above the exciton resonance.

We have shown in Ref. [5] that for electron trapping mediated by the bound trion resonance, the optimal experimental conditions correspond to tuning the laser slightly below the exciton resonance and above the trion resonance. In this configuration, trapping potentials with depths of 1 to 2 meV can be achieved with a relatively low laser intensity (a few kW/cm$^2$ in typical III-V or II-VI materials), in order to avoid sizeable laser heating. The Pauli-mediated trapping calculated in this letter has an overall reduction factor $(a_X/a_T)^D$ compared to the trion mediated trapping, $a_T$ being the characteristic trion size. It however has a more favorable detuning dependence: for large detunings, the Pauli-mediated optical trapping scales as $\sim \Omega^2/E^X$, while the bound trion mediated trapping scales as $\sim \Omega^2\Delta^T/(E^X)^2$, where $\Delta^T$ is the trion binding energy. Pauli-mediated trapping depths comparable to the bound trion case can in fact be obtained by using stronger laser intensity but larger detuning so as to avoid heating effects. Such a regime of strong field and large detuning precisely is the one typically used for atomic optical lattices.

**Conclusion.** – We have shown that counter-propagating laser beams used in a semiconductor system can trap electrons in optical lattices similar to the ones of atomic systems. In contrast to other theoretical schemes existing in the literature [5,8], we here demonstrated that Pauli blocking is sufficient to confine carriers if the laser polarization and electron spin allow for the excitation of a virtual electron pair in a triplet state. This configuration which does not take advantage of the trionic enhancement effects discussed in Ref. [5], shows that optical trapping of electrons in semicondutors can be realized using quite different spin channels. Moreover, the Pauli-assisted optical potential scales, like for atoms, as the inverse detuning in the limit of lasers well below the excitonic resonances, in contrast to the inverse detuning square dependence of the trion-assisted optical potential. We also show that excitonic effects can lead to both an attractive and repulsive Pauli-assisted potential, depending if the laser frequency is tuned above or below the ground state exciton resonance. The scheme discussed here opens to many possibilities for integrating optical and electronic control in semiconductor systems, with potential applications to research areas such as Bose-Einstein condensation of excitons, quantum information and spintronics.

* * *

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