Nagy,

Tibor

Keep this exam CLOSED until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, student number (PID). Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your test and your bubble sheet.

Thank you and good luck!

Possibly useful constant:

- $g = 9.81 \text{ m/s}^2$

Possibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$
Please, sit in row G.

1 pt Are you sitting in the seat assigned?

☐ No, I am not.

☐ Yes, I am.
There are 149 steps between the ground floor and the sixth floor in a building. Each step is 17.1 cm tall. It takes 2 minutes and 33 seconds for a person with a mass of 68.8 kg to walk all the way up. How much work did the person do?

(work)

What was the average power performed by the person during the walk?

(power)

Number of steps: \( n = 149 \)

Height of one step: \( h = 17.1 \text{ cm} = 0.171 \text{ m} \)

Total height: \( H = n \cdot h \)

Mass of the person: \( m = 68.8 \text{ kg} \)

Work done by the person: \( W = mg \cdot H = mg \cdot nh \approx 17,200 \text{ J} \)

Time of the walk: \( \Delta t = 2\text{ min } 33\text{ sec} = 153 \text{ s} \)

Power of the person: \( P = \frac{W}{\Delta t} = \frac{17,200}{153} \approx 112 \text{ W} \)
An airplane is flying with a speed of 247 km/h at a height of 4000 m above the ground. A parachutist whose mass is 93.4 kg, jumps out of the airplane, opens the parachute and then lands on the ground with a speed of 3.30 m/s. How much energy was dissipated on the parachute by the air friction?

\( \text{(in MJ)} \)

4. \( \text{A} \) 3.04 \( \text{B} \) 3.44 \( \text{C} \) 3.88 \( \text{D} \) 4.39 \( \text{E} \) 4.96 \( \text{F} \) 5.60 \( \text{G} \) 6.33 \( \text{H} \) 7.16

\[
\begin{align*}
\text{Energy balance:} \\
KE_i + PE_i &= KE_f + \Delta E_{th} \\
\frac{1}{2}mv_i^2 + mgh &= \frac{1}{2}mv_f^2 + \Delta E_{th} \\
\Delta E_{th} &= \frac{1}{2}m(v_i^2 - v_f^2) + mgh = \\
&= \frac{1}{2} \cdot 93.4 \cdot \left( \left( \frac{247}{3.6} \right)^2 - 3.3^2 \right) + 93.4 \cdot 9.81 \cdot 4000 = \\
&= 3.88 \text{ MJ}
\end{align*}
\]
By what percent does the braking distance of a car increase, when the speed of the car increases by 18.9 percent? Braking distance is the distance a car travels from the point when the brakes are applied to when the car comes to a complete stop.

\[
\text{Braking: } KE_i + W_{\text{diss}} = 0
\]
\[
KE_i = -W_{\text{diss}}
\]
\[
\frac{1}{2} m v_i^2 = F \cdot d
\]
\[
v_i^2 \propto d
\]

18.9\% increase \implies increase by a factor of 1.189

\[1.189^2 \approx 1.414 \implies 41.4\% \text{ increase}\]
A railroad cart with a mass of $m_1 = 13.4 \text{ t}$ is at rest at the top of an $h = 11.8 \text{ m}$ high hump yard hill. It is pushed very slowly over the edge, it starts to roll down. At the bottom it hits another cart originally at rest with a mass of $m_2 = 17.0 \text{ t}$. The bumper mechanism locks the two carts together. What is the final common speed of the two carts? (Neglect losses due to rolling friction of the carts. The letter t stands for metric ton in the SI system.)

\[ \sqrt{2gh} = \frac{1}{2} m_1 u_1^2 \]

Collision: conservation of momentum:

\[ m_1 \cdot v_1 + m_2 \cdot 0 = (m_1 + m_2) \cdot v_f \]

\[ \frac{m_1}{m_1 + m_2} \cdot v_1 = v_f \]

\[ \frac{m_1}{m_1 + m_2} \cdot \sqrt{2gh} = v_f \]

\[ v_f = \frac{13.4}{13.4 + 17.0} \cdot \sqrt{2 \cdot 9.81 \cdot 11.8} \]

\[ v_f = 6.71 \text{ m/s} \]
The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.

This function can be described by the following formula:
\[ x(t) = A \cos(\omega t), \]
where \( x \) and \( A \) are measured in meters, \( t \) is measured in seconds, \( \omega \) is measured in rad/s.

Using the graph determine the angular frequency \( \omega \) of the oscillation.

\[
\omega = \frac{2\pi}{T} = \frac{6.28}{4} = 1.57 \text{ rad/s}
\]
4 pt An object with a mass of \( m = 1.27 \text{ kg} \) connected to a spring oscillates on a horizontal frictionless surface as shown in the figure.

The equation of the motion of the mass is given by

\[
x(t) = A \cdot \cos(\omega t)
\]

where the position \( x \) is measured in meters, the time \( t \) is measured in seconds. Determine the total mechanical energy of the mass spring oscillator.

\( \text{(in J)} \)

\[
\begin{align*}
\text{A} & : 6.59 \times 10^{-2} \\
\text{B} & : 8.77 \times 10^{-2} \\
\text{C} & : 1.17 \times 10^{-1} \\
\text{D} & : 1.55 \times 10^{-1} \\
\text{E} & : 2.06 \times 10^{-1} \\
\text{F} & : 2.74 \times 10^{-1} \\
\text{G} & : 3.65 \times 10^{-1} \\
\text{H} & : 4.85 \times 10^{-1}
\end{align*}
\]

\[
\begin{align*}
\text{TE} &= KE_{\text{max}} \quad (= PE_{\text{max}}) \\
KE_{\text{max}} &= \frac{1}{2} m \nu_{\text{max}}^2 \\
\nu_{\text{max}} &= A \omega \\
\text{Everything combined together:} \\
\text{TE} &= KE_{\text{max}} = \frac{1}{2} m (A \omega)^2 \\
\text{TE} &= \frac{1}{2} \cdot 1.27 \cdot (0.319 \cdot 1.01)^2 = 6.59 \times 10^{-2} \text{ J}
\end{align*}
\]
An extended body (not shown in the figure) has its center of mass (CM) at the origin of the reference frame. In the case below give the direction for the torque \( \tau \) with respect to the CM on the body due to force \( F \) acting on the body at a location indicated by the vector \( r \).

Torque: \( \tau = \vec{r} \times \vec{F} \)

RHR:
- 1st finger (thumb)
- 2nd finger (pointer)
- 3rd finger (middle finger)
Three small objects are located in the x-y plane as shown in the figure. All three objects have the same mass, \( m = 1.77 \text{ kg} \).

What is the moment of inertia of this set of objects with respect to the axis perpendicular to the x-y plane passing through location \( x = 3.00 \text{ m} \) and \( y = 3.00 \text{ m} \)? (The objects are small in size, their moments of inertia about their own centers of mass are negligibly small.)

(in kg\( \cdot \text{m}^2 \))

\[
I = m r_A^2 + m r_B^2 + m r_C^2 = m (r_A^2 + r_B^2 + r_C^2) = 1.77 \cdot (8 + 17 + 29) = 1.77 \cdot 54 = 95.58 \text{ kg} \cdot \text{m}^2
\]
A solid, homogeneous cylinder with mass of $M = 2.75$ kg and a radius of $R = 18.3$ cm is resting at the top of an incline as shown in the figure.

The height of the incline is $h = 1.49$ m, and the angle of the incline is $\theta = 14.3^\circ$. The cylinder is rolled over the edge very slowly. Then it rolls down to the bottom of the incline without slipping. What is the final speed of the cylinder?

\[ \text{(in m/s)} \]

11.  
A 1.45  
B 1.81  
C 2.26  
D 2.83  
E 3.53  
F 4.41  
G 5.52  
H 6.90

**Final translational speed after rolling down from an incline of height $h$:**

\[ v_{\text{t,f}} = \sqrt{\frac{2gh}{1+k}} = \sqrt{\frac{2 \cdot 9.81 \cdot 1.49}{1 + 0.5}} \]

\[ v_{\text{t,f}} = 4.41 \text{ m/s} \]
2 pt  You ride your bicycle in the forward direction on a straight horizontal road. What is the direction of the velocity vector of your bicycle?

12. A○ up to the sky  
   B○ forward  
   C○ to your right  
   D○ down to the ground  
   E○ backward  
   F○ The velocity is zero.  
   G○ to your left

2 pt  What is the direction of the angular velocity vector of your wheels?

13. A○ backward  
   B○ to your left  
   C○ to your right  
   D○ forward  
   E○ down to the ground  
   F○ up to the sky  
   G○ The angular velocity is zero.

\[ \mathbf{\omega} \] use the curled finger right hand rule.
A crate with a mass of $M = 72.5\ \text{kg}$ is suspended by a rope from the endpoint of a uniform boom. The boom has a mass of $m = 126\ \text{kg}$ and a length of $l = 8.02\ \text{m}$. The midpoint of the boom is supported by another rope which is horizontal and is attached to the wall as shown in the figure.

The boom makes an angle of $\theta = 51.6^\circ$ with the vertical wall. Calculate the tension in the vertical rope.

\[ \text{Vertical rope: } Mg = 72.5 \cdot 9.81 = 711\ \text{N} \]

What is the tension in the horizontal rope?

\[ \text{Horizontal rope: } (2M+m)g \tan \theta = T \]

\[ T = (2 \cdot 72.5 + 126) \cdot 9.81 \cdot \tan 51.6^\circ \]

\[ T = 3354\ \text{N} \]