Nagy,

Tibor

Keep this exam **CLOSED** until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, **student number (PID)**. Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your **test** and your **bubble sheet**.

Thank you and good luck!

Posssibly useful constant:

- $g = 9.81$ m/s^2

Posssibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$

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Please, sit in row G.

$\boxed{\textit{1 pt}}$ Are you sitting in the seat assigned?

1.A◯ Yes, I am.

$\boxed{3\ pt}$ There are 149 steps between the ground floor and the sixth floor in a building. Each step is 17.1 cm tall. It takes 2 minutes and 33 seconds for a person with a mass of 68.8 kg to walk all the way up. How much work did the person do?
(in J)

2. **A**◯ 1.28×10^3 **B**◯ 1.85×10^3 **C**◯ 2.68×10^3 **D**◯ 3.89×10^3
E◯ 5.64×10^3 **F**◯ 8.18×10^3 **G**◯ 1.19×10^4 **H**◯ 1.72×10^4

$\boxed{3\ pt}$ What was the average power performed by the person during the walk?
(in W)

3. **A**◯ 3.74×10^1 **B**◯ 4.38×10^1 **C**◯ 5.13×10^1 **D**◯ 6.00×10^1
E◯ 7.02×10^1 **F**◯ 8.21×10^1 **G**◯ 9.61×10^1 **H**◯ 1.12×10^2

Number of steps: $n = 149$.
Height of one step: $h = 17.1\,cm = 0.171\,m$.
Total height: $H = n \cdot h$
Mass of the person: $m = 68.8\,kg$

Work done by the person:

$$W = mg \cdot H = mg \cdot nh \cong 17{,}200\,J$$

Time of the walk:

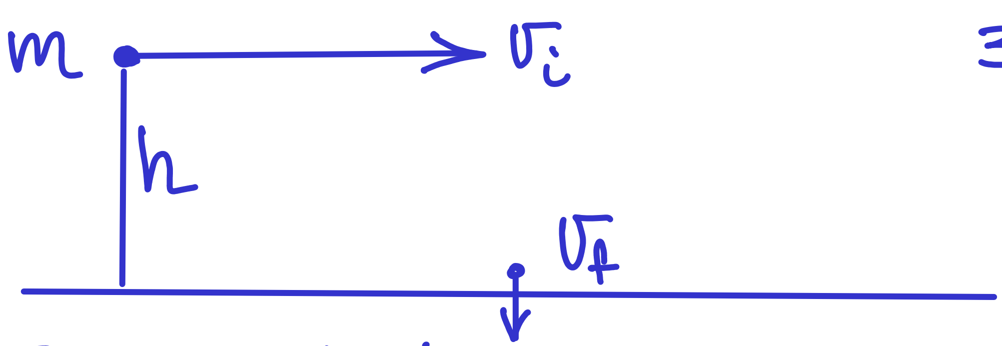
$$\Delta t = 2\,min\,33\,sec = 153\,s$$

Power of the person:

$$P = \frac{W}{\Delta t} = \frac{17{,}200}{153} = 112\,W$$

$\boxed{4\ pt}$ An airplane is flying with a speed of 247 km/h at a height of 4000 m above the ground. A parachutist whose mass is 93.4 kg, jumps out of the airplane, opens the parachute and then lands on the ground with a speed of 3.30 m/s. How much energy was dissipated on the parachute by the air friction?
(in MJ*)*

4. **A**◯ 3.04 **B**◯ 3.44 **C**◯ 3.88 **D**◯ 4.39 **E**◯ 4.96 **F**◯ 5.60 **G**◯ 6.33 **H**◯ 7.16



$$3.6\,\frac{km}{h} = 1.0\,\frac{m}{s}$$

Energy balance:

$$KE_i + PE_i = KE_f + \Delta E_{th}$$

$$\frac{1}{2} m v_i^2 + mgh - \frac{1}{2} m v_f^2 = \Delta E_{th}$$

$$\Delta E_{th} = \frac{1}{2} m \left(v_i^2 - v_f^2 \right) + mgh =$$

$$= \frac{1}{2} \cdot 93.4 \cdot \left(\left(\frac{247}{3.6} \right)^2 - 3.3^2 \right) + 93.4 \cdot 9.81 \cdot 4000 =$$

$$= 3.88\ MJ$$

$\boxed{4\ pt}$ By what percent does the braking distance of a car increase, when the speed of the car increases by 18.9 percent? Braking distance is the distance a car travels from the point when the brakes are applied to when the car comes to a complete stop.

5. **A**◯ 7.47 **B**◯ 9.94 **C**◯ 1.32×10^1 **D**◯ 1.76×10^1
E◯ 2.34×10^1 **F**◯ 3.11×10^1 **G**◯ 4.14×10^1 **H**◯ 5.50×10^1

Braking: $KE_i + W_{diss} = 0$

$$KE_i = -W_{diss}$$

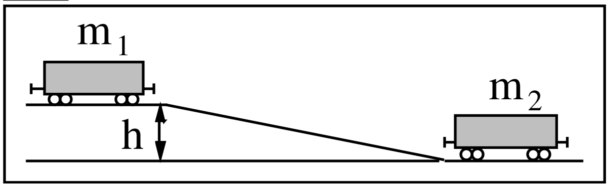
$$\frac{1}{2} m v_i^2 = F \cdot d$$

$$v_i^2 \propto d$$

18.9% increase → increase by a factor of 1.189

$1.189^2 \cong 1.414$ → 41.4% increase

5 pt A railroad cart with a mass of $m_1 = 13.4$ t is at rest at the top of an $h = 11.8$ m high hump yard hill.



After it is pushed very slowly over the edge, it starts to roll down. At the bottom it hits another cart originally at rest with a mass of $m_2 = 17.0$ t. The bumper mechanism locks the two carts together. What is the final common speed of the two carts? (Neglect losses due to rolling friction of the carts. The letter t stands for metric ton in the SI system.) *(in* m/s*)*

6. **A** ◯ 6.71 **B** ◯ 8.92 **C** ◯ 1.19×10^1 **D** ◯ 1.58×10^1
E ◯ 2.10×10^1 **F** ◯ 2.79×10^1 **G** ◯ 3.71×10^1 **H** ◯ 4.94×10^1

Cart m_1 rolls down: conservation of energy: $m_1 g h = \frac{1}{2} m_1 v_1^2$

$$\sqrt{2gh} = v_1$$

Collision: conservation of momentum:

$$m_1 \cdot v_1 + m_2 \cdot 0 = (m_1 + m_2) \cdot v_f$$

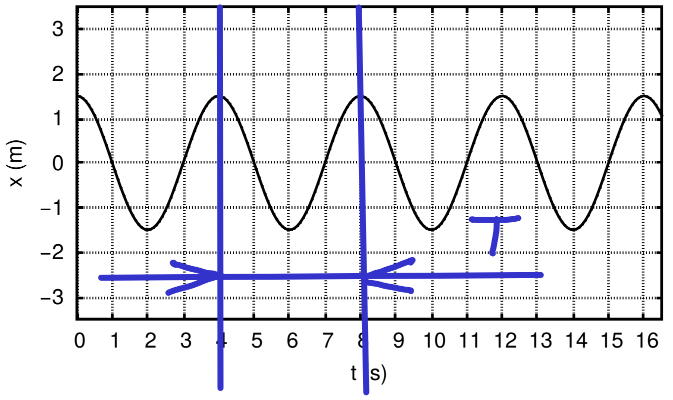
$$\frac{m_1}{m_1 + m_2} \cdot v_1 = v_f$$

$$\frac{m_1}{m_1 + m_2} \cdot \sqrt{2gh} = v_f$$

$$v_f = \frac{13.4}{13.4 + 17.0} \cdot \sqrt{2 \cdot 9.81 \cdot 11.8}$$

$$v_f = 6.71 \text{ m/s}$$

The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.



T: period

T = 4s

This function can be described by the following formula:
x(t) = Acos(ωt), where x and A are measured in meters, t is measured in seconds, ω is measured in rad/s.

$\boxed{4\ pt}$ Using the graph determine the angular frequency ω of the oscillation.
(in `rad/s`*)*

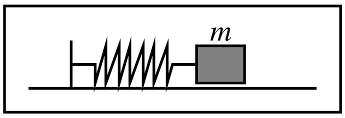
7. **A**◯ 2.45×10^{-1} **B**◯ 3.55×10^{-1} **C**◯ 5.15×10^{-1} **D**◯ 7.47×10^{-1}
E◯ 1.08 **F**◯ 1.57 **G**◯ 2.28 **H**◯ 3.30

Angular frequency :

$$\omega = \frac{2\pi}{T} = \frac{6.28}{4} = 1.57\ \frac{rad}{s}$$

$\boxed{4\ pt}$ An object with a mass of $m = 1.27$ kg connected to a spring oscillates on a horizontal frictionless surface as shown in the figure.

Simple harmonic oscill:
$x(t) = A \cdot \cos(\omega t)$



The equation of the motion of the mass is given by
$x = 0.319 \cos(1.01t)$
where the position x is measured in meters, the time t is measured in seconds. Determine the total mechanical energy of the mass spring oscillator.
(in J*)*

8. **A**◯ 6.59×10^{-2} **B**◯ 8.77×10^{-2} **C**◯ 1.17×10^{-1} **D**◯ 1.55×10^{-1}
E◯ 2.06×10^{-1} **F**◯ 2.74×10^{-1} **G**◯ 3.65×10^{-1} **H**◯ 4.85×10^{-1}

$TE = KE_{max} \quad (= PE_{max})$

$KE_{max} = \frac{1}{2} m v_{max}^2$

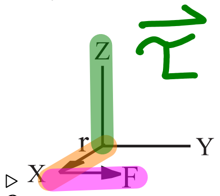
$v_{max} = A\omega$

Everything combined together:

$TE = KE_{max} = \frac{1}{2} m (A\omega)^2$

$TE = \frac{1}{2} \cdot 1.27 \cdot (0.319 \cdot 1.01)^2 = 6.59 \cdot 10^{-2}$ J

$\boxed{\textit{4 pt}}$ An extended body (not shown in the figure) has its center of mass (CM) at the origin of the reference frame. In the case below give the direction for the torque τ with respect to the CM on the body due to force **F** acting on the body at a location indicated by the vector **r**.



9. **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

Torque : $\vec{\tau} = \vec{r} \times \vec{F}$

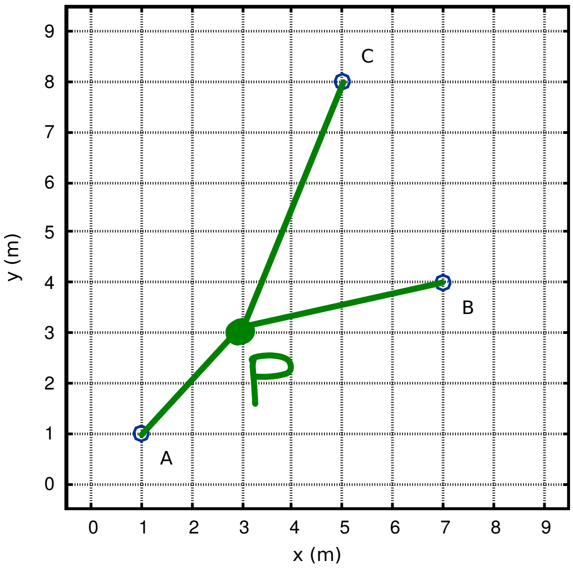
RHR :

2nd finger (pointer)

1st finger (thumb)

3rd finger (middle finger)

4 pt Three small objects are located in the x-y plane as shown in the figure. All three objects have the same mass, m = 1.77 kg.



r^2

A: $2^2 + 2^2 = 8$

B: $4^2 + 1^2 = 17$

C: $2^2 + 5^2 = 29$

What is the moment of inertia of this set of objects with respect to the axis perpendicular to the the x-y plane passing through location x = 3.00 m and y = 3.00 m? (The objects are small in size, their moments of inertia about their own centers of mass are negligibly small.)
(in `kg*m^2`*)*

10. **A**◯ 6.62×10^1 **B**◯ 7.49×10^1 **C**◯ 8.46×10^1 **D**◯ 9.56×10^1
E◯ 1.08×10^2 **F**◯ 1.22×10^2 **G**◯ 1.38×10^2 **H**◯ 1.56×10^2

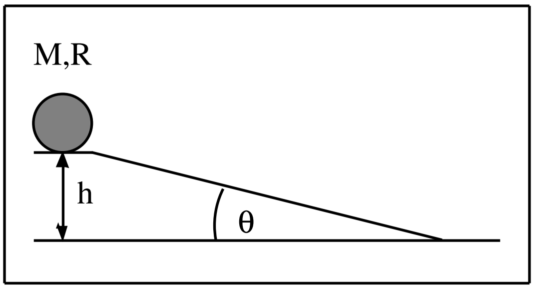
$$I = m r_A^2 + m r_B^2 + m_c^2 =$$

$$= m(r_A^2 + r_B^2 + r_c^2) =$$

$$= 1.77 \cdot (8 + 17 + 29) =$$

$$= 1.77 \cdot 54 = 95.58 \text{ kgm}^2$$

$\boxed{\textit{4 pt}}$ A solid, homogeneous cylinder with of mass of M = 2.75 kg and a radius of R = 18.3 cm is resting at the top of an incline as shown in the figure.



Solid cylinder:

$$I = \underbrace{\frac{1}{2}}_{k} MR^2$$

The height of the incline is h = 1.49 m, and the angle of the incline is $\theta = 14.3°$. The cylinder is rolled over the edge very slowly. Then it rolls down to the bottom of the incline without slipping. What is the final speed of the cylinder? *(in* `m/s`*)*

11. **A**◯ 1.45 **B**◯ 1.81 **C**◯ 2.26 **D**◯ 2.83 **E**◯ 3.53 **F**◯ 4.41 **G**◯ 5.52 **H**◯ 6.90

Final translational speed after rolling down from an incline of height h:

$$v_{t,f} = \sqrt{\frac{2gh}{1+k}} = \sqrt{\frac{2 \cdot 9.81 \cdot 1.49}{1 + 0.5}}$$

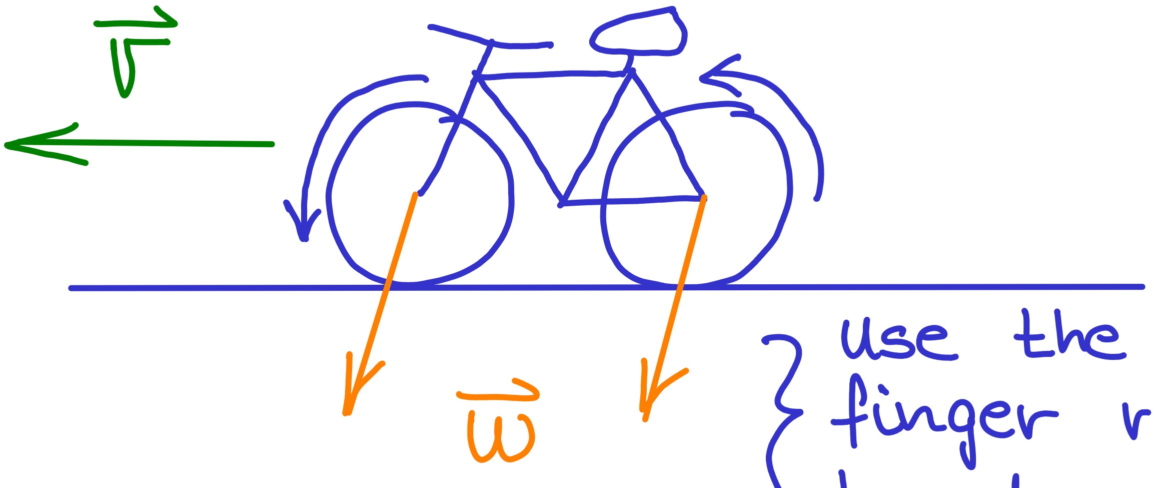
$$v_{t,f} = 4.41 \text{ m/s}$$

$\boxed{\textit{2 pt}}$ You ride your bicycle in the forward direction on a straight horizontal road. What is the direction of the velocity vector of your bicycle?

12.A◯ up to the sky
B◯ forward
C◯ to your right
D◯ down to the ground
E◯ backward
F◯ The velocity is zero.
G◯ to your left

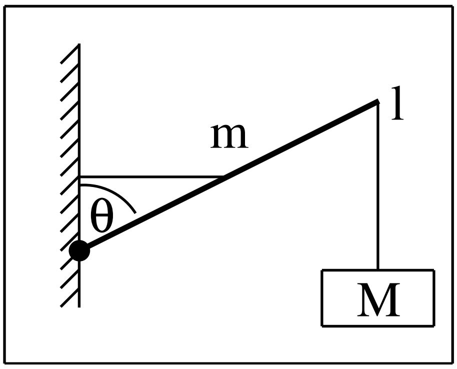
$\boxed{\textit{2 pt}}$ What is the direction of the angular velocity vector of your wheels?

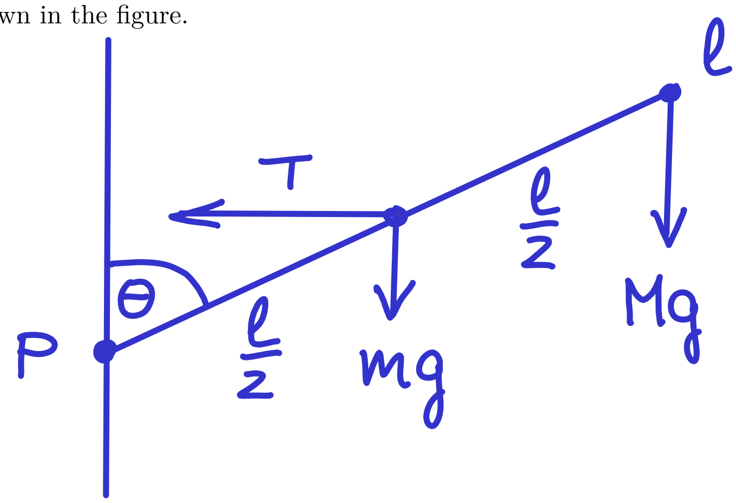
13.A◯ backward
B◯ to your left
C◯ to your right
D◯ forward
E◯ down to the ground
F◯ up to the sky
G◯ The angular velocity is zero.



Use the curled finger right hand rule.

A crate with a mass of M = 72.5 kg is suspended by a rope from the endpoint of a uniform boom. The boom has a mass of m = 126 kg and a length of l = 8.02 m. The midpoint of the boom is supported by another rope which is horizontal and is attached to the wall as shown in the figure.





3 pt The boom makes an angle of $\theta = 51.6°$ with the vertical wall. Calculate the tension in the vertical rope. *(in N)*

Vertical rope: $Mg = 72.5 \cdot 9.81 =$
$= 711$ N

14. **A**◯ 7.11×10^2 **B**◯ 8.89×10^2 **C**◯ 1.11×10^3 **D**◯ 1.39×10^3
E◯ 1.74×10^3 **F**◯ 2.17×10^3 **G**◯ 2.71×10^3 **H**◯ 3.39×10^3

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3 pt What is the tension in the horizontal rope? *(in N)*

15. **A**◯ 5.23×10^2 **B**◯ 7.59×10^2 **C**◯ 1.10×10^3 **D**◯ 1.60×10^3
E◯ 2.31×10^3 **F**◯ 3.35×10^3 **G**◯ 4.86×10^3 **H**◯ 7.05×10^3

Pivot point: P
Balance of torques:

$$\underbrace{Mg\,l\sin\theta + mg\frac{l}{2}\sin\theta}_{\text{clockwise torques}} = \underbrace{T\cdot\frac{l}{2}\cos\theta}_{\text{ccw. torque}}$$

$$(2M+m)\,g\tan\theta = T$$

$$T = (2\cdot 72.5 + 126)\cdot 9.81\cdot \tan 51.6°$$

$$T = 3354\text{ N}$$