Nagy,

Tibor

Keep this exam CLOSED until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, student number (PID). Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your test and your bubble sheet.

Thank you and good luck!

Possibly useful constant:

- \( g = 9.81 \text{ m/s}^2 \)

Possibly useful Moments of Inertia:

- Solid homogeneous cylinder: \( I_{CM} = \frac{1}{2}MR^2 \)
- Solid homogeneous sphere: \( I_{CM} = \frac{2}{5}MR^2 \)
- Thin spherical shell: \( I_{CM} = \frac{2}{3}MR^2 \)
- Thin uniform rod, axis perpendicular to length: \( I_{CM} = \frac{1}{12}ML^2 \)
- Thin uniform rod around end, axis perpendicular to length: \( I_{end} = \frac{1}{3}ML^2 \)
Please, sit in row G.

1 pt Are you sitting in the seat assigned?

1. A☐ Yes, I am.
A crane at a construction site lifts a concrete block with a mass of \( m = 442 \text{ kg} \) from the ground to the top of a building with a height of \( h = 15.9 \text{ m} \). How much work did the motor of the crane do? (Assume zero energy loss in the lifting mechanism due to friction.) 

\[
(W_{\text{in J}}) = \text{mg} \cdot h
\]

\[
W = 442 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 15.9 \text{ m}
\]

\[
W = 6.89 \times 10^4 \text{ J}
\]

\[
P = \frac{W}{\Delta t} = \frac{6.89 \times 10^4 \text{ J}}{60 \cdot 2.90 \text{ s}} = 396 \text{ W}
\]
An ideal spring is fixed at one end. A variable force $\mathbf{F}$ pulls on the spring. When the magnitude of $\mathbf{F}$ reaches a value of 31.9 N, the spring is stretched by 16.0 cm from its equilibrium length. Calculate the additional work required by $\mathbf{F}$ to stretch the spring by an additional 10.3 cm from that position.

\[
W_{\text{add}} = \frac{F_1 + F_2}{2} \cdot (\Delta x_2 - \Delta x_1) = \frac{31.9 + 52.4}{2} \cdot 0.103 \quad \text{J}
\]
A 631 kg automobile slides across an icy street at a speed of 41.2 km/h and collides with a parked car which has a mass of 985 kg. The two cars lock up and slide together. What is the speed of the two cars just after they collide? (in km/h)

\[ \begin{array}{cccc}
A & 1.61 \times 10^1 & B & 2.01 \times 10^1 \\
C & 2.51 \times 10^1 & D & 3.14 \times 10^1 \\
E & 3.93 \times 10^1 & F & 4.91 \times 10^1 \\
G & 6.14 \times 10^1 & H & 7.67 \times 10^1 
\end{array} \]

\[
\frac{m_1 \cdot \vec{v}_i + m_2 \cdot 0}{(m_1 + m_2)} = \vec{v}_f
\]

\[
\frac{m_1}{m_1 + m_2} \cdot \vec{v}_i = \vec{v}_f
\]

\[
\vec{v}_f = \frac{631}{631 + 985} \cdot 41.2 = 16.1 \text{ km/h}
\]

Do not convert from km/h to m/s, it's not necessary.
A 62.5 kg wood board is resting on very smooth ice in the middle of a frozen lake. A 48.5 kg boy stands at one end of the board. He walks from one end of the board to the other end with a velocity of 1.91 m/s relative to the ice in the positive direction. What is the velocity of the board relative to the ice?

\[
\text{Velocity of the board relative to the ice} = \frac{m_b \cdot v_b}{m_b + m_l}
\]

\[
\text{Velocity of the board relative to the ice} = \frac{48.5 \cdot 1.91}{62.5} = 1.48 \text{ m/s}
\]

\[
\text{Velocity of the board relative to the ice} = -1.48 \text{ m/s}
\]
3 pt Select True or False for the following statement.

The center of mass of an extended object is always located within the material of the object.

7. A ☐ True  B ☐ False

The center of mass of a ring is not part of the ring:

Center of Mass
The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.

This function can be described by the following formula:
\[ x(t) = A \sin(\omega t) \]
where \( x \) and \( A \) are measured in meters, \( t \) is measured in seconds, \( \omega \) is measured in rad/s.

**3 pt** Using the graph determine the amplitude \( A \) of the oscillation. 

\[ A = 2.1 \text{ m} \]

8.  A ○ 1.20  B ○ 1.80  C ○ 2.10  D ○ 2.40  E ○ 3.60  F ○ 4.20  G ○ 4.50  H ○ 4.80

**3 pt** Determine the period \( T \) of the oscillation. 

\[ T = 3.6 \text{ s} \]

9.  A ○ 2.80  B ○ 3.60  C ○ 4.00  D ○ 5.20  E ○ 5.60  F ○ 6.40  G ○ 6.80  H ○ 7.60

**3 pt** If the mass of the object is 48.5 g, then what is the total mechanical energy of this oscillator? 

\[ \text{TE} = \frac{1}{2} k A^2 \]

\[ k = m \omega^2 \]

\[ \omega = \frac{2\pi}{T} \]

\[ \text{TE} = \frac{1}{2} m \left( \frac{2\pi}{T} \right)^2 \cdot A^2 = \frac{1}{2} \cdot 0.0485 \cdot \left( \frac{6.28}{3.60} \right)^2 \cdot 2.1^2 = 0.326 \text{ J} \]

Notice: \( \text{TE} = \text{PE}_{\text{max}} = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} m v_{\text{max}}^2 = KE_{\text{max}} \)
An extended body (not shown in the figure) has its center of mass (CM) at the origin of the reference frame. In the case below give the direction for the torque $\mathbf{\tau}$ with respect to the CM on the body due to force $\mathbf{F}$ acting on the body at a location indicated by the vector $\mathbf{r}$.

$\mathbf{\tau} = \mathbf{r} \times \mathbf{F}$

11. A X  B -X  C Y  D -Y  E Z  F -Z
Three small objects are located in the x-y plane as shown in the figure. All three objects have the same mass, \( m = 1.99 \text{ kg} \).

What is the moment of inertia of this set of objects with respect to the axis perpendicular to the x-y plane passing through location \( x = 5.00 \text{ m} \) and \( y = 4.00 \text{ m} \)? (The objects are small in size, their moments of inertia about their own centers of mass are negligibly small.)

\[
I = I_A + I_B + I_C = m r_A^2 + m r_B^2 + m r_C^2 = m (r_A^2 + r_B^2 + r_C^2)
\]

\[
r_A^2 = 4^2 + 3^2 = 25
\]
\[
r_B^2 = 1^2 + 1^2 = 2
\]
\[
r_C^2 = 0^2 + 2^2 = 4
\]

\[\sqrt{2^2 + 3^2} = 3.1 \text{ m}^2\]

\[I = 1.99 \text{ kg} \cdot 31 \text{ m}^2 = 61.7 \text{ kgm}^2\]
3 pt You are going to back out of a driveway. You put your car in reverse and you start driving backwards. The driveway is straight and horizontal. What is the direction of the velocity vector of your car?

13. A\(\_\_\_\) to your left  
   B\(\_\_\_\) up to the sky  
   C\(\_\_\_\) down to the ground  
   D\(\_\_\_\) to your right  
   E\(\_\_\_\) backward  
   F\(\_\_\_\) The velocity is zero.  
   G\(\_\_\_\) forward

3 pt What is the direction of the angular velocity vector of your wheels?

14. A\(\_\_\_\) backward  
   B\(\_\_\_\) The angular velocity is zero.  
   C\(\_\_\_\) to your left  
   D\(\_\_\_\) to your right  
   E\(\_\_\_\) down to the ground  
   F\(\_\_\_\) up to the sky  
   G\(\_\_\_\) forward
An object with a mass of \( m = 119 \text{ kg} \) is suspended by a rope from the end of a uniform boom with a mass of \( M = 67.3 \text{ kg} \) and a length of \( l = 7.77 \text{ m} \). The end of the boom is supported by another rope which is horizontal and attached to the wall as shown in the figure.

The boom makes an angle of \( \theta = 58.3^\circ \) with the vertical wall. Calculate the tension in the vertical rope.

\[
T = \left( \frac{M}{2} + m \right) \cdot g \cdot \tan \theta
\]

\[
T = \left( \frac{67.3}{2} + 119 \right) \cdot 9.81 \cdot \tan 58.3^\circ
\]

\[
T = 2425 \text{ N}
\]