Nagy,

Tibor

Keep this exam **CLOSED** until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, **student number (PID)**. Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your **test** and your **bubble sheet**.

Thank you and good luck!

Posssibly useful constant:

- $g = 9.81$ m/s^2

Posssibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$

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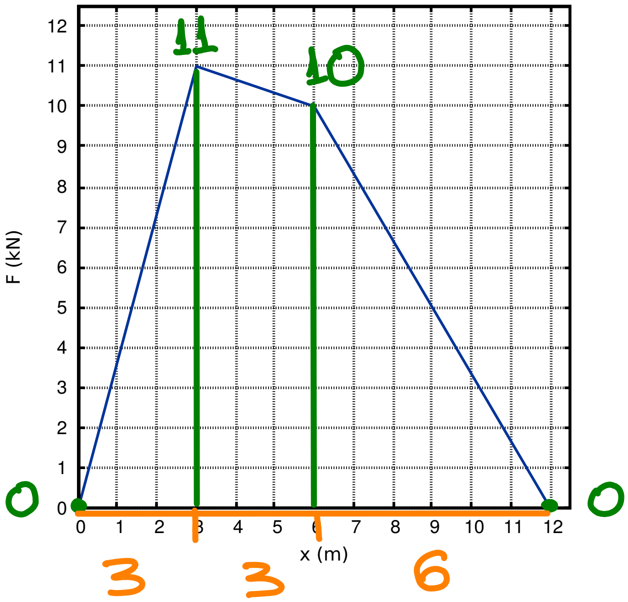
Please, sit in row I.

$\boxed{\textit{1 pt}}$ Are you sitting in the seat assigned?

1.A ◯ Yes, I am.

A lawn mower tractor pulls a large and heavy sled during a pulling game at a county fair. The force during the pull is monitored and recorded by instruments. The figure shows the force as a function of displacement.



The area under the force vs displacement graph is work.

Work divided by the time the work took is power.

3 pt What was the total amount of work done by the tractor?
(in kJ*)*

2. **A**◯ 30.4 **B**◯ 35.6 **C**◯ 41.6 **D**◯ 48.7 **E**◯ 57.0 **F**◯ 66.7 **G**◯ 78.0 **H**◯ 91.3

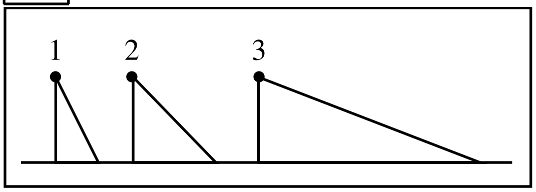
3 pt What was the average power of the tractor, if the pull lasted for 8.60 seconds?
(in kW*)*

3. **A**◯ 2.18 **B**◯ 2.90 **C**◯ 3.86 **D**◯ 5.13
E◯ 6.82 **F**◯ 9.07 **G**◯ 12.06 **H**◯ 16.04

$$W = \frac{0+11}{2}\cdot 3 + \frac{11+10}{2}\cdot 3 + \frac{10+0}{2}\cdot 6 = 78\text{ kJ}$$

$$P = \frac{W}{\Delta t} = \frac{78\text{ kJ}}{8.6\text{ s}} = 9.07\text{ kW}$$

3 pt Three small identical masses are released simultaneously from the top of three inclines. (See figure.)



The three inclines have the same height and they are all frictionless, but they have different angles. Which object will have the highest speed at the bottom of the incline?

4.A◯ All three objects will have the same speed at the bottom.
B◯ Object #3 will have the highest speed.
C◯ Object #2 will have the highest speed.
D◯ Object #1 will have the highest speed.

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3 pt Which object will finish the race first?

5.A◯ Object #2 will finish first.
B◯ Object #3 will finish first.
C◯ All three objects will finish at the same time.
D◯ Object #1 will finish first.

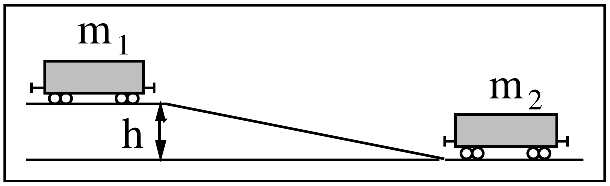
As the three objects slide down on the inclines, they convert their potential energy to kinetic energy:

$$PE_i = KE_f$$
$$mgh_i = \frac{1}{2}mv_f^2$$
$$2gh_i = v_f^2$$

Same height means same speed at the bottom.

The object on the steepest incline will win the race, because it has the largest acceleration and the shortest distance to cover.

5 pt A railroad cart with a mass of $m_1 = 12.2$ t is at rest at the top of an $h = 11.4$ m high hump yard hill.



After it is pushed very slowly over the edge, it starts to roll down. At the bottom it hits another cart originally at rest with a mass of $m_2 = 23.2$ t. The bumper mechanism locks the two carts together. What is the final common speed of the two carts? (Neglect losses due to rolling friction of the carts. The letter t stands for metric ton in the SI system.) *(in* m/s*)*

6. **A**◯ 1.65 **B**◯ 2.19 **C**◯ 2.91 **D**◯ 3.88
E◯ 5.15 **F**◯ 6.86 **G**◯ 9.12 **H**◯ 1.21×10^1

First cart rolling down:

$$PE_i = KE_f$$

$$m_1 g h = \frac{1}{2} m_1 v_1^2$$

$$\sqrt{2gh} = v_1$$

The collision between the carts is perfectly inelastic, because the carts lock together.

Conservation of momentum:

$$m_1 \cdot v_1 + m_2 \cdot 0 = (m_1 + m_2) \cdot v_{common}$$

$$\frac{m_1}{m_1 + m_2} \cdot v_1 = v_{common}$$

$$\frac{m_1}{m_1 + m_2} \cdot \sqrt{2gh} = v_{common}$$

(1t = 1000 kg, but we don't need it.)

4 pt A 811 kg automobile is sliding on an icy street. It collides with a parked car which has a mass of 623 kg. The two cars lock up and slide together with a speed of 17.3 km/h. What was the speed of the first car just before the collision?
(*in* `km/h`)

7. **A**◯ 2.40×10^1 **B**◯ 2.71×10^1 **C**◯ 3.06×10^1 **D**◯ 3.46×10^1
E◯ 3.91×10^1 **F**◯ 4.41×10^1 **G**◯ 4.99×10^1 **H**◯ 5.64×10^1

Since the two cars lock together, the collision is perfectly inelastic. In any inelastic collision – perfectly inelastic or partially (in)elastic collision – the energy (kinetic energy) is not conserved. Only momentum is conserved:

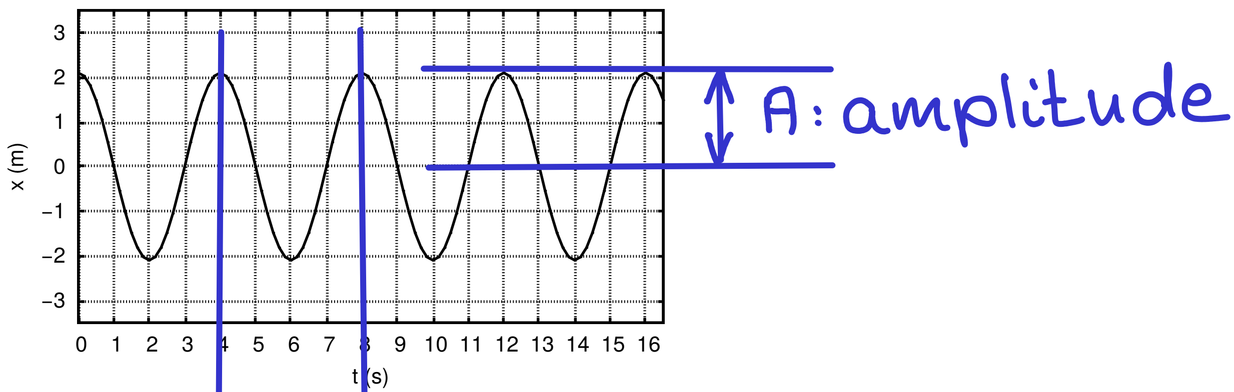
$$m_1 v_1 + m_2 \cdot 0 = (m_1 + m_2) v_f$$

$$v_1 = \frac{m_1 + m_2}{m_1} \cdot v_f$$

$$v_1 = \frac{811 + 623}{811} \cdot 17.3$$

$$v_1 = 30.6 \text{ km/h}$$

The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.



This function can be described by the following formula:
x(t) = Acos(ωt), where x and A are measured in meters, t is measured in seconds, ω is measured in rad/s.

T: period

$\boxed{3\ pt}$ Using the graph determine the amplitude A of the oscillation.
(in m)

8. **A**◯ 2.10 **B**◯ 2.40 **C**◯ 2.70 **D**◯ 3.00 **E**◯ 3.60 **F**◯ 3.90 **G**◯ 4.50 **H**◯ 4.80

$\boxed{3\ pt}$ Determine the period T of the oscillation.
(in s)

9. **A**◯ 2.40 **B**◯ 3.20 **C**◯ 4.00 **D**◯ 5.20 **E**◯ 6.00 **F**◯ 6.80 **G**◯ 7.20 **H**◯ 7.60

4 pt The period of a mass-spring oscillator is 2.08 s. Every time the oscillator completes a full period, the amplitude of the oscillation gets reduced to 94.4 percent of the previous amplitude. How much time does it take for the amplitude to decay to 49.7 percent of its original initial value? *(in* s*)*

10. **A**◯ 1.55×10^1 **B**◯ 1.75×10^1 **C**◯ 1.98×10^1 **D**◯ 2.23×10^1
E◯ 2.52×10^1 **F**◯ 2.85×10^1 **G**◯ 3.22×10^1 **H**◯ 3.64×10^1

$A_{n+1} = f \cdot A_n$ where $f = 94.4\% = 0.944$

Exponential decay:

$$0.944^n = 0.497$$

$$n \cdot \ln 0.944 = \ln 0.497$$

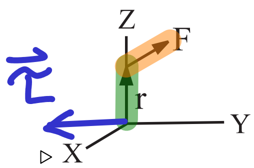
$$n = \frac{\ln 0.497}{\ln 0.944} = 12.13 \text{ periods}$$

$$t = n \cdot T = 12.13 \cdot 2.08 = 25.23\text{ s}$$

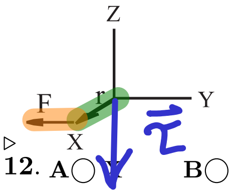
$\boxed{\textit{6 pt}}$ A body (not shown) has its center of mass (CM) at the origin. In each case below give the direction for the torque τ with respect to the CM on the body due to force **F** acting on the body at a location indicated by the vector **r**.

Torque:

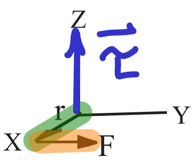
$\vec{\tau} = \vec{r} \times \vec{F}$



▷ **11.** **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

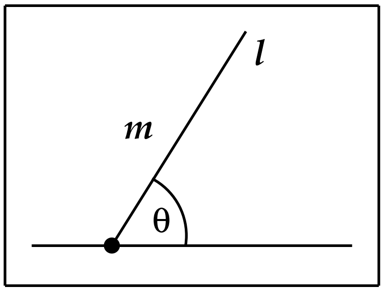


▷ **12.** **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z



▷ **13.** **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

A uniform rod with a mass of $m = 1.78$ kg and a length of $l = 2.26$ m is attached to a horizontal surface with a hinge. The rod can rotate around the hinge without friction. (See figure.)



The rod is held at rest at an angle of $\theta = 67.5°$ with respect to the horizontal surface.

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3 pt What is the angular acceleration of the rod, when it is released?
(in `rad/s^2`*)*

14. **A** ◯ 1.33 **B** ◯ 1.56 **C** ◯ 1.82 **D** ◯ 2.13 **E** ◯ 2.49 **F** ◯ 2.92 **G** ◯ 3.41 **H** ◯ 3.99

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3 pt What is the angular speed of the rod, when it hits the horizontal surface?
(in `rad/s`*)*

15. **A** ◯ 2.72 **B** ◯ 3.07 **C** ◯ 3.47 **D** ◯ 3.92 **E** ◯ 4.43 **F** ◯ 5.00 **G** ◯ 5.66 **H** ◯ 6.39

Newton's second law for rotation:

$$\tau = I\alpha$$

$$mg\frac{l}{2}\cos\theta = \frac{1}{3}ml^2 \cdot \alpha$$

$$\frac{3}{2} \cdot \frac{g}{l} \cdot \cos\theta = \alpha$$

Conservation of energy:

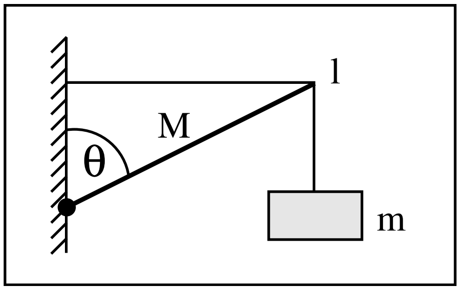
$$PE_i = KE_{r,f}$$

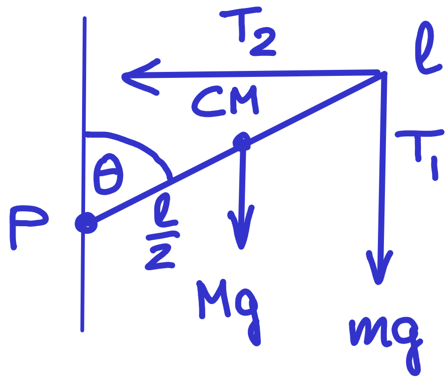
$$mgh = \frac{1}{2}I\omega_f^2$$

$$mg\frac{l}{2}\sin\theta = \frac{1}{2} \cdot \frac{1}{3}ml^2\omega_f^2$$

$$3\frac{g}{l}\sin\theta = \omega_f^2$$

An object with a mass of m = 103 kg is suspended by a rope from the end of a uniform boom with a mass of M = 79.5 kg and a length of l = 9.50 m. The end of the boom is supported by another rope which is horizontal and attached to the wall as shown in the figure.





3 pt The boom makes an angle of $\theta = 62.2°$ with the vertical wall. Calculate the tension in the vertical rope.
(in N*)*

16. **A**◯ 3.23×10^2 **B**◯ 4.29×10^2 **C**◯ 5.71×10^2 **D**◯ 7.60×10^2
E◯ 1.01×10^3 **F**◯ 1.34×10^3 **G**◯ 1.79×10^3 **H**◯ 2.38×10^3

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3 pt Calculate the tension in the horizontal rope. (The horizontal and the vertical ropes are not connected to each other. They are both independently attached to the end of the boom.)
(in N*)*

17. **A**◯ 2.66×10^3 **B**◯ 3.53×10^3 **C**◯ 4.70×10^3 **D**◯ 6.25×10^3
E◯ 8.31×10^3 **F**◯ 1.11×10^4 **G**◯ 1.47×10^4 **H**◯ 1.96×10^4

Vertical rope : $T_1 = mg$

Horizontal rope : T_2

Torque balance wrt pivot P :

$$Mg\frac{\ell}{2}\sin\theta + mg\ell\sin\theta = T_2\ell\cos\theta$$

$$\left(\frac{Mg}{2} + mg\right)\cdot\tan\theta = T_2$$