Nagy,

Tibor

Keep this exam CLOSED until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, student number (PID). Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your test and your bubble sheet.

Thank you and good luck!

Possibly useful constant:

- $g = 9.81 \text{ m/s}^2$

Possibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$
Please, sit in row I.

1 pt Are you sitting in the seat assigned?

1. A Yes, I am.
An athlete, swimming at a constant speed, covers a distance of 137 m in a time period of 1.95 minutes. The drag force exerted by the water on the swimmer is 50.0 N. Calculate the power the swimmer must provide in overcoming that force.

\( P = F \cdot \frac{d}{t} \)

\[ \text{Power: } P = \vec{F} \cdot \vec{v} = F \cdot v \]

\[ \text{since } \theta = 0^\circ \]
\[ \text{and } \cos(0^\circ) = 1 \]

\[ F = 50 \text{ N} \] \text{ and } \[ v = \frac{d}{t} \]

\[ P = F \cdot \frac{d}{t} = 50 \text{ N} \cdot \frac{137 \text{ m}}{1.95 \text{ min} \cdot 60 \text{ s/min}} \]

\[ P = 58.5 \text{ W} \]
A point mass \( m = 180 \text{ g} \) hangs from a string of length \( l = 1.20 \text{ m} \).

\[ h = l - l \cdot \cos \theta = l (1 - \cos \theta) \]

**Conservation of energy:**

\[
\frac{1}{2} m v^2 = mgh
\]

\[
v = \sqrt{2gh} = \sqrt{2g l (1 - \cos \theta)}
\]

\[
v = \sqrt{2 \cdot 9.81 \cdot 1.2 \cdot (1 - \cos(34.4^\circ))} = 2.03 \text{ m/s}
\]
What happens to the braking distance when a car’s speed increases by 20 percent? (Braking distance is the distance a car will travel from the point when the brakes are applied to when the car comes to a complete stop.)

4. A. It increases by 44 percent.
   B. It increases by 40 percent.
   C. It increases by 21 percent.
   D. Nothing, it remains the same.
   E. It increases by 46 percent.
   F. It increases by 42 percent.
   G. It increases by 20 percent.
   H. It increases by 48 percent.

\[ KE = \frac{1}{2} mv^2: \text{ kinetic energy is quadratic in speed. In the braking process the kinetic energy is turned to thermal energy: } KE_i = \Delta E_{th} \]

20% increase means a factor of 1.2 increase. \(1.2^2 = 1.44\) which is an increase of 44%.
4 pt An automobile is sliding across an icy street at a speed of 65.9 km/h and it collides with a parked car. The two cars lock up and they slide together with a speed of 32.5 km/h. If the mass of the parked car is 1220 kg, then what is the mass of the first car? (in kg)

5. A \(2.14 \times 10^2\)  B \(2.85 \times 10^2\)  C \(3.79 \times 10^2\)  D \(5.05 \times 10^2\)  
E \(6.71 \times 10^2\)  F \(8.93 \times 10^2\)  G \(1.19 \times 10^3\)  H \(1.58 \times 10^3\)

Momentum conservation:

\[ m_1 \cdot v_i + m_2 \cdot 0 = (m_1 + m_2) \cdot v_f \]

\[ m_1 v_i = m_1 v_f + m_2 v_f \]

\[ m_1 v_i - m_1 v_f = m_2 v_f \]

\[ m_1 (v_i - v_f) = m_2 v_f \]

\[ m_1 = m_2 \cdot \frac{v_f}{v_i - v_f} \]

\[ m_1 = 1220 \cdot \frac{32.5}{65.9 - 32.5} \]

\[ m_1 = 1187 \text{ kg} \]

Keep the speeds in km/h, don’t convert to m/s. The km/h units will cancel out in the last fraction.
The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.

This function can be described by the following formula:

\[ x(t) = A \sin(\omega t) \]

where \( x \) and \( A \) are measured in meters, \( t \) is measured in seconds, \( \omega \) is measured in rad/s.

Using the graph determine the amplitude \( A \) of the oscillation.

6. \( A \) 4.00 \times 10^{-1} \quad B \quad 1.30 \quad C \quad 1.60 \quad D \quad 1.90
   E \quad 2.20 \quad F \quad 3.70 \quad G \quad 4.30 \quad H \quad 4.60

Determine the period \( T \) of the oscillation.

7. \( A \) 4.60 \quad B \quad 5.40 \quad C \quad 5.80 \quad D \quad 7.00 \quad E \quad 8.20 \quad F \quad 8.60 \quad G \quad 9.40 \quad H \quad 9.80
An object is performing simple harmonic motion. The maximum value of the object’s speed is 1.79 m/s and the maximum value of its acceleration is 8.62 m/s$^2$. Determine the amplitude of the motion. (in m)

8.  

- A $4.00 \times 10^{-2}$
- B $5.80 \times 10^{-2}$
- C $8.41 \times 10^{-2}$
- D $1.22 \times 10^{-1}$
- E $1.77 \times 10^{-1}$
- F $2.56 \times 10^{-1}$
- G $3.72 \times 10^{-1}$
- H $5.39 \times 10^{-1}$

Determine the angular frequency of the motion. (in 1/s)

9.  

- A $4.82$
- B $5.63$
- C $6.59$
- D $7.71$
- E $9.02$
- F $1.06 \times 10^1$
- G $1.24 \times 10^1$
- H $1.45 \times 10^1$

Maximum speed: $v_{\text{max}} = Aw$

Maximum acceleration: $a_{\text{max}} = Aw^2$

\[
\frac{a_{\text{max}}}{v_{\text{max}}} = \frac{Aw^2}{Aw} = \omega
\]

\[
\omega = \frac{a_{\text{max}}}{v_{\text{max}}} = \frac{8.62 \text{ m/s}^2}{1.79 \text{ m/s}} = 4.82 \text{ 1/s}
\]

$v_{\text{max}} = Aw \Rightarrow A = \frac{v_{\text{max}}}{\omega}$

\[
A = \frac{1.79 \text{ m/s}}{4.82 \text{ 1/s}} = 0.372 \text{ m}
\]

\[
A = \frac{v_{\text{max}}^2}{\omega} \Rightarrow a_{\text{max}} = \frac{v_{\text{max}}^2}{A}
\]

\[
a_{\text{cp}} = \frac{v^2}{R}
\]
A body (not shown) has its center of mass (CM) at the origin. In each case below give the direction for the torque $\tau$ with respect to the CM on the body due to force $\mathbf{F}$ acting on the body at a location indicated by the vector $\mathbf{r}$.

10. $\mathbf{F}$ acts at point B.

11. $\mathbf{F}$ acts at point C.

12. $\mathbf{F}$ acts at point D.

Torque: $\mathbf{\tau} = \mathbf{r} \times \mathbf{F}$

You use the right hand rule.
A uniform rod with a mass of $m = 1.98$ kg and a length of $l = 2.34$ m is attached to a horizontal surface with a hinge. The rod can rotate around the hinge without friction. (See figure.)

The rod is held at rest at an angle of $\theta = 72.7^\circ$ with respect to the horizontal surface. What is the angular acceleration of the rod just before it lands on the horizontal surface? (in rad/s$^{-2}$)

Newton’s second law for rotation:

$$\tau = I_p \cdot \alpha$$

$$mg \frac{l}{2} = \frac{1}{3} ml^2 \cdot \alpha$$

$$\frac{3}{2} \frac{9}{l} = \alpha$$

$$\alpha = \frac{3}{2} \cdot \frac{9.8 \text{ m/s}^2}{2.34 \text{ m}} = 6.29 \text{ rad/s}^2$$
A spherical shell with a mass of $M = 2.35$ kg and a radius of $R = 13.9$ cm is resting at the top of an incline as shown in the figure.

\[
\begin{align*}
\text{Spherical shell:} \\
I_{CM} &= \frac{2}{3} MR^2
\end{align*}
\]

The height of the incline is $h = 1.75$ m, and the angle of the incline is $\theta = 15.5^\circ$. The spherical shell is rolled over the edge very slowly. Then it rolls down to the bottom of the incline without slipping. What is the final speed of the shell?

(in m/s)

\[
\begin{array}{cccc}
\text{A} & 4.54 & \text{B} & 5.13 \\
\text{C} & 5.80 & \text{D} & 6.55 \\
\text{E} & 7.40 & \text{F} & 8.36 \\
\text{G} & 9.45 & \text{H} & 10.68
\end{array}
\]

\[\text{Conservation of energy:}
PE_i = KE_{t,f} + KE_{r,f}
\]

\[Mgh = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2
\]

\[Mgh = \frac{1}{2} M v_f^2 + \frac{1}{2} \cdot \frac{2}{3} \cdot MR^2 \omega_f^2
\]

\[2gh = v_f^2 + \frac{2}{3} v_f^2 \quad \Rightarrow \quad R \omega_f = v_f : \text{no slip}
\]

\[\sqrt{\frac{2gh}{1 + \frac{2}{3}}} = v_f
\]

\[v_f = \sqrt{\frac{2g}{\frac{5}{3}}} h = \sqrt{\frac{2 \cdot 9.81 \cdot 1.75}{\frac{5}{3}}} = 4.54 \text{ m/s}
\]
An object with a mass of $m = 112$ kg is suspended by a rope from the end of a uniform boom with a mass of $M = 58.9$ kg and a length of $l = 8.77$ m. The end of the boom is supported by another rope which is horizontal and attached to the wall as shown in the figure.

\[ mg = 112 \times 9.81 \times \frac{m}{g^2} = 1099 \text{ N} \]

### 15. The boom makes an angle of $\theta = 69.1^\circ$ with the vertical wall. Calculate the tension in the vertical rope.

<table>
<thead>
<tr>
<th>Option</th>
<th>Tension (in N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$8.15 \times 10^2$</td>
</tr>
<tr>
<td>B</td>
<td>$1.18 \times 10^2$</td>
</tr>
<tr>
<td>C</td>
<td>$7.17 \times 10^2$</td>
</tr>
<tr>
<td>D</td>
<td>$2.49 \times 10^2$</td>
</tr>
<tr>
<td>E</td>
<td>$3.60 \times 10^2$</td>
</tr>
<tr>
<td>F</td>
<td>$5.23 \times 10^2$</td>
</tr>
<tr>
<td>G</td>
<td>$7.58 \times 10^2$</td>
</tr>
<tr>
<td>H</td>
<td>$1.10 \times 10^2$</td>
</tr>
</tbody>
</table>

### 16. Calculate the tension in the horizontal rope. (The horizontal and the vertical ropes are not connected to each other. They are both independently attached to the end of the boom.)

<table>
<thead>
<tr>
<th>Option</th>
<th>Tension (in N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2.73 \times 10^3$</td>
</tr>
<tr>
<td>B</td>
<td>$3.63 \times 10^3$</td>
</tr>
<tr>
<td>C</td>
<td>$4.83 \times 10^3$</td>
</tr>
<tr>
<td>D</td>
<td>$6.43 \times 10^3$</td>
</tr>
<tr>
<td>E</td>
<td>$8.55 \times 10^3$</td>
</tr>
<tr>
<td>F</td>
<td>$1.14 \times 10^4$</td>
</tr>
<tr>
<td>G</td>
<td>$1.51 \times 10^4$</td>
</tr>
<tr>
<td>H</td>
<td>$2.01 \times 10^4$</td>
</tr>
</tbody>
</table>

### Balance of the torques at about $P$:

\[
Mg \frac{l}{2} \sin \theta + mg l \sin \theta = T \cdot l \cos \theta
\]

\[
\left( \frac{l}{2} M + m \right) g \tan \theta = T
\]

\[
T = (\frac{1}{2} 58.9 + 112) \cdot 9.81 \cdot \tan (69.1^\circ)
\]

\[
T = 3634 \text{ N}
\]