Nagy,

Tibor

Keep this exam **CLOSED** until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, **student number (PID)**. Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your **test** and your **bubble sheet**.

Thank you and good luck!

Posssibly useful constant:

- $g = 9.81\ m/s^2$

Posssibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$

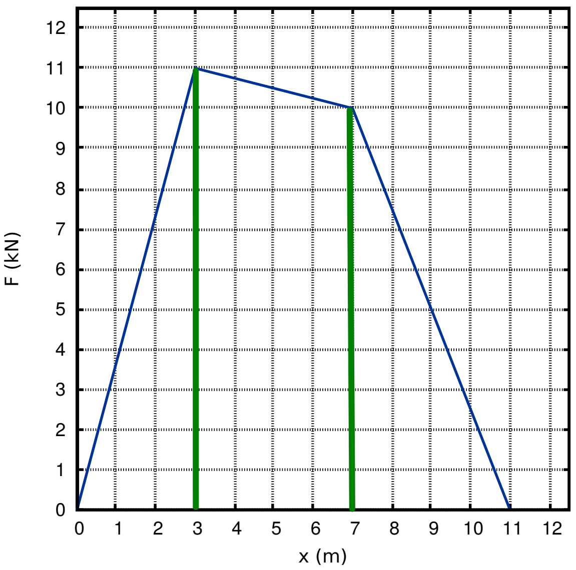
nagytibo@msu

Please, sit in row D.

1 pt Are you sitting in the seat assigned?

1.A ◯ Yes, I am.

A lawn mower tractor pulls a large and heavy sled during a pulling game at a county fair. The force during the pull is monitored and recorded by instruments. The figure shows the force as a function of displacement.



The area under the force vs. displacement function is work. Work divided by time is power.

2 pt What was the total amount of work done by the tractor?
(in kJ*)*

2. **A**◯ 44.4 **B**◯ 59.0 **C**◯ 78.5 **D**◯ 104.4
E◯ 138.9 **F**◯ 184.7 **G**◯ 245.6 **H**◯ 326.7

---

2 pt What was the average power of the tractor, if the pull lasted for 8.30 seconds?
(in kW*)*

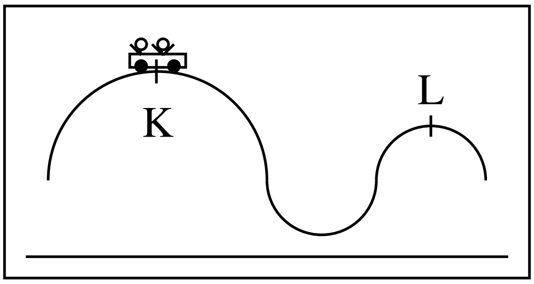
3. **A**◯ 5.80 **B**◯ 6.55 **C**◯ 7.41 **D**◯ 8.37
E◯ 9.46 **F**◯ 10.69 **G**◯ 12.08 **H**◯ 13.65

$$W = \frac{3 \cdot 11}{2} + \frac{11+10}{2} \cdot 4 + \frac{4 \cdot 10}{2} = 16.5 + 42 + 20$$

$W = 78.5$ kJ (b/c m · kN = kJ)

$$P = \frac{78.5\ \text{kJ}}{8.3\ \text{s}} = 9.46\ \text{kW} \quad (\text{b/c } \frac{\text{kJ}}{\text{s}} = \text{kW})$$

On a roller coaster ride the total mass of a cart - with two passengers included - is 285 kg. Peak **K** is at 49.8 m above the ground and peak **L** is at 22.4 m. At location **K** the speed of the cart is 17.0 m/s, and at location **L** it is 12.6 m/s. (The wheel mechanism on roller coaster carts always keeps the carts safely on the rail.)



4 pt How much mechanical energy is lost due to friction between the two peaks?
(in J)

4. **A**◯ 5.94×10^4 **B**◯ 6.95×10^4 **C**◯ 8.13×10^4 **D**◯ 9.52×10^4
E◯ 1.11×10^5 **F**◯ 1.30×10^5 **G**◯ 1.52×10^5 **H**◯ 1.78×10^5

Energy balance:

$$KE_K + PE_K = KE_L + PE_L + \Delta E_{th}$$

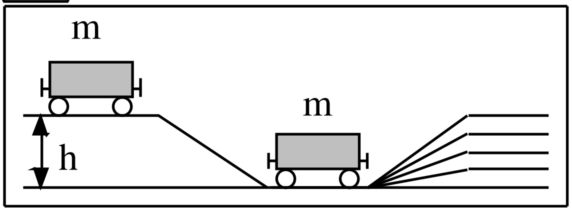
$$\frac{1}{2}mv_K^2 + mgh_K - \frac{1}{2}mv_L^2 - mgh_L = \Delta E_{th}$$

$$m\left[\frac{1}{2}(v_K^2 - v_L^2) + g(h_K - h_L)\right] = \Delta E_{th}$$

$$285\left[\frac{1}{2}(17.0^2 - 12.6^2) + 9.81\cdot(49.8 - 22.4)\right] = \Delta E_{th}$$

$$\Delta E_{th} = 9.52\cdot 10^4 \text{ J}$$

4 pt A railroad cart with mass **m** is at rest on the top of a hill with height **h**. (See figure.)



Then it starts to roll down. At the bottom it collides with an identical cart. The two carts lock together. How high can they reach together? (Neglect any losses due to friction.)

5.A◯ **h**, the original height.
B◯ (1/2)**h**, half of the original height.
C◯ (1/4)**h**, one quarter of the original height.
D◯ Zero, they cannot climb any height.
E◯ (3/4)**h**, three quarter of the original height.

→ Cart #1 rolls down: conservation of energy: $PE_i = KE_f \Rightarrow mgh = \frac{1}{2}mv^2$
$\Rightarrow 2gh = v^2$

→ Collision: conservation of momentum
$m v_i = (2m) \cdot v_f \Rightarrow \frac{v_i}{2} = v_f$

→ Compound system climbs: conservation of energy: $v_f^2 = 2gh_f$.
Since $v_f = \frac{v_i}{2}$ therefore $h_f = \frac{h_i}{4} = \frac{h}{4}$

Kinetic energy: $KE = \frac{1}{2}mv^2$
Potential energy: $PE = mgh$
} If you double your speed, you can climb four times higher. If you halve your speed, you can climb only one quarter of the original height.

4 pt A 743 kg automobile slides across an icy street at a speed of 53.1 km/h and collides with a parked car. The two cars lock up and they slide together with a speed of 25.9 km/h. What is the mass of the parked car?
(*in* kg)

6. **A**◯ 8.40×10^1 **B**◯ 1.22×10^2 **C**◯ 1.77×10^2 **D**◯ 2.56×10^2
E◯ 3.71×10^2 **F**◯ 5.38×10^2 **G**◯ 7.80×10^2 **H**◯ 1.13×10^3

Conservation of momentum:

$$m_1 v_i + m_2 \cdot 0 = (m_1 + m_2) v_f$$

$$m_1 v_i = m_1 v_f + m_2 v_f$$

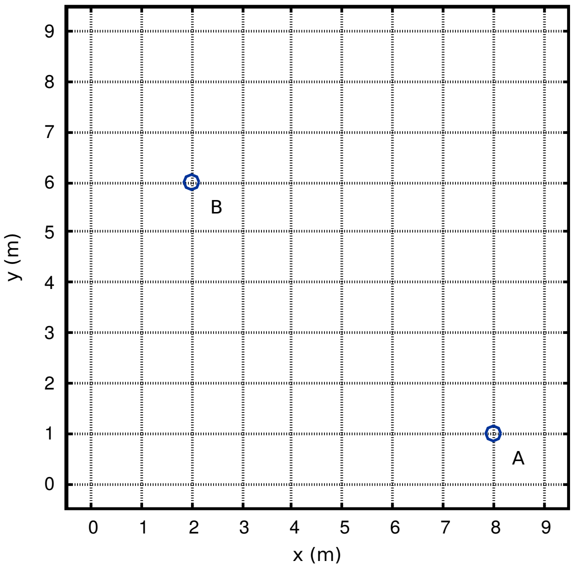
$$m_1 v_i - m_1 v_f = m_2 v_f$$

$$m_1 \frac{v_i - v_f}{v_f} = m_2$$

$$m_2 = 743 \cdot \frac{53.1 - 25.9}{25.9} = 780\,\text{kg}$$

Keep the speeds in km/h units, don't convert to m/s. The km/h units will cancel in the division.

4 pt Two small but dense objects are located in the x-y plane as shown in the figure. The objects have the following masses: $m_A = 3.59$ kg, $m_B = 2.35$ kg.



Center of mass:

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

(for point masses)

Determine the x-coordinate of the center of the mass of this system. The objects are small in size, they can be treated as point masses.
(in m*)*

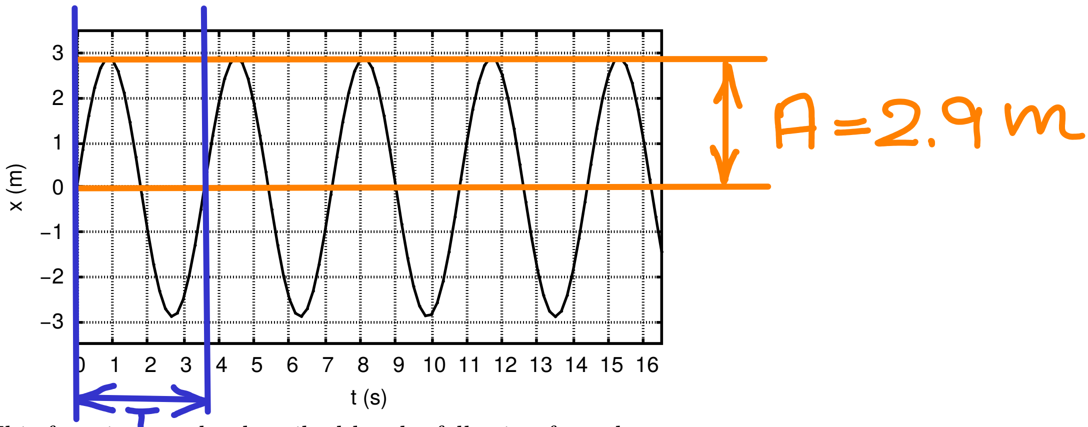
7. **A**◯ 2.19 **B**◯ 2.57 **C**◯ 3.00 **D**◯ 3.51 **E**◯ 4.11 **F**◯ 4.81 **G**◯ 5.63 **H**◯ 6.58

X-coordinate of the CM:

$$x_{CM} = \frac{m_A \cdot x_A + m_B x_B}{m_A + m_B} =$$

$$= \frac{3.59 \cdot 8 + 2.35 \cdot 2}{3.59 + 2.35} = 5.63\,m$$

The graph shows the x-displacement as a function of time for a particular object undergoing simple harmonic motion.



This function can be described by the following formula:
x(t) = Asin(ωt), where x and A are measured in meters, t is measured in seconds, ω is measured in rad/s.

T=3.6s

---

2 pt Using the graph determine the amplitude A of the oscillation.
(in m)

8. **A** ◯ 2.90 **B** ◯ 3.50 **C** ◯ 4.70 **D** ◯ 5.00 **E** ◯ 5.30 **F** ◯ 5.60 **G** ◯ 5.90 **H** ◯ 6.20

---

2 pt Determine the period T of the oscillation.
(in s)

9. **A** ◯ 2.00 **B** ◯ 2.80 **C** ◯ 3.20 **D** ◯ 3.60 **E** ◯ 4.80 **F** ◯ 6.00 **G** ◯ 6.80 **H** ◯ 7.60

4 pt Due to a technical malfunction a space explorer had to crash land on Planet-X. She manages to fix her space ship and now she is preparing for launch. However she needs to know the gravitational acceleration on the surface of the planet in order to take off successfully. She builds a mathematical pendulum out of a piece of string and a left over steel bolt. The bolt has a mass of 48.7 g, and the string is 163 cm long. She attaches the pendulum to a fixed point and she lets it swing. She counts 14 complete oscillations in a time period of 58.7 seconds. What is the gravitational acceleration on the surface of Planet-X?
(in `m/s^2`*)*

10. **A**◯ 1.43 **B**◯ 1.67 **C**◯ 1.95 **D**◯ 2.29 **E**◯ 2.67 **F**◯ 3.13 **G**◯ 3.66 **H**◯ 4.28

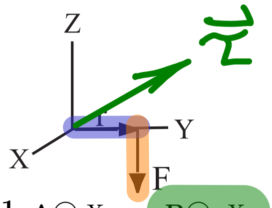
$14T = 58.7s \Rightarrow T = 4.19s$

Period of a mathematical pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T^2}{4\pi^2} = \frac{l}{g} \Rightarrow$$

$$\Rightarrow g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 \cdot 1.63}{4.19^2} = 3.66\ \frac{m}{s^2}$$

3 pt An extended body (not shown in the figure) has its center of mass (CM) at the origin of the reference frame. In the case below give the direction for the torque τ with respect to the CM on the body due to force **F** acting on the body at a location indicated by the vector **r**.



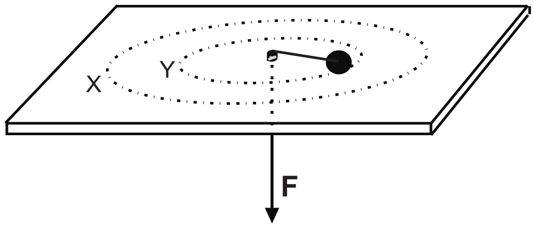
Torque: $\vec{\tau} = \vec{r} \times \vec{F}$

3rd finger ($\vec{\tau}$), 1st finger ($\vec{r}$), 2nd finger ($\vec{F}$)

▷ **11.** **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

Use the right hand rule.

10 pt A small mass M attached to a string slides in a circle (Y) on a frictionless horizontal table, with the force **F** providing the necessary tension (see figure). The force is then decreased slowly and then maintained constant when M travels around in circle (X). The radius of circle (X) is twice the radius of circle (Y).



▷ M's angular momentum at X is .... that at Y.
12. **A**◯ true **B**◯ false **C**◯ greater than **D**◯ less than **E**◯ equal to

▷ M's angular velocity at Y is four times that at X.
13. **A**◯ true **B**◯ false **C**◯ greater than **D**◯ less than **E**◯ equal to

▷ M's kinetic energy at X is half that at Y.
14. **A**◯ true **B**◯ false **C**◯ greater than **D**◯ less than **E**◯ equal to

▷ While going from Y to X, there is no torque on M.
15. **A**◯ true **B**◯ false **C**◯ greater than **D**◯ less than **E**◯ equal to

▷ As M moves from Y to X, the work done by **F** is .... 0.
16. **A**◯ true **B**◯ false **C**◯ greater than **D**◯ less than **E**◯ equal to

---

→ $W<0$ b/c force F and displacement d points in the opporite directions.

→ $\tau=0$ b/c the force F is central.

→ $L_X=L_Y$ b/c $\tau=0$ $\left(\tau=\frac{\Delta L}{\Delta t}\right)$

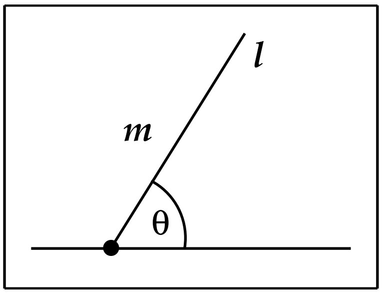
→ $\omega_X=\frac{1}{4}\omega_Y$ from $\omega=\frac{L}{I}=\frac{L}{mr^2}$

$\omega \propto \frac{1}{r^2}$

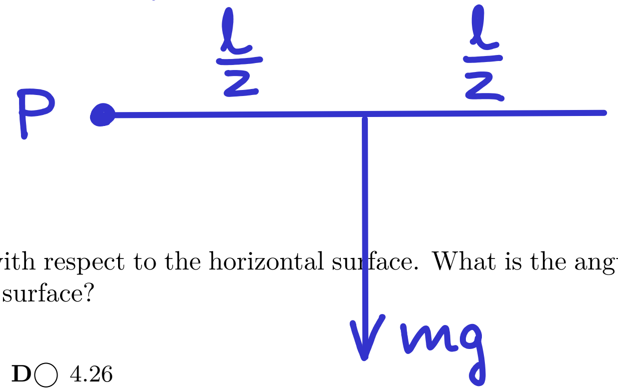
→ $KE_X=\frac{1}{4}KE_Y$ from $KE=\frac{L^2}{2I}=\frac{L^2}{2mr^2}$

$KE \propto \frac{1}{r^2}$

4 pt A uniform rod with a mass of $m = 1.98$ kg and a length of $l = 2.38$ m is attached to a horizontal surface with a hinge. The rod can rotate around the hinge without friction. (See figure.)



just before the rod lands:



The rod is held at rest at an angle of $\theta = 66.7°$ with respect to the horizontal surface. What is the angular acceleration of the rod just before it lands on the horizontal surface?
(in `rad/s^2`*)*

17. **A** ◯ 1.40 **B** ◯ 2.03 **C** ◯ 2.94 **D** ◯ 4.26
E ◯ 6.18 **F** ◯ 8.97 **G** ◯ 13.00 **H** ◯ 18.85

Newton's 2nd law for rotations:

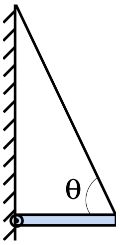
$\tau = I_P \cdot \alpha$ where $I_P = \frac{1}{3} m l^2$

$$mg \cdot \frac{l}{2} = \frac{1}{3} m l^2 \cdot \alpha$$

$$\frac{3}{2} \cdot \frac{g}{l} = \alpha$$

$$\alpha = \frac{3}{2} \cdot \frac{9.81\ m/s^2}{2.38\ m} = 6.18\ rad/s^2.$$

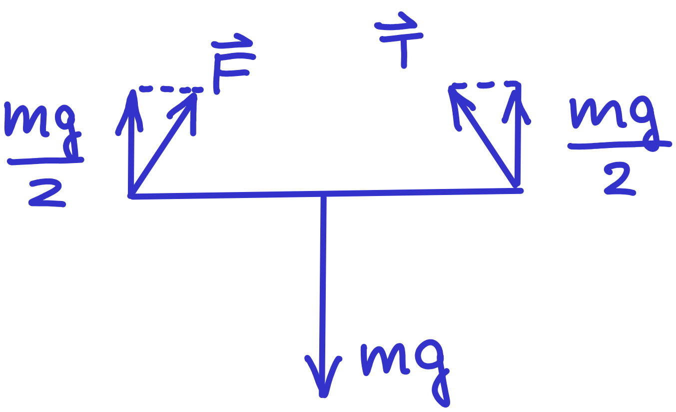
4 pt A 27.0 kg beam is attached to a wall with a hinge while its far end is supported by a cable such that the beam is horizontal.



If the angle between the beam and the cable is $\theta = 70.0°$ what is the vertical component of the force exerted by the hinge on the beam?
(in N*)*

18. **A** ◯ 1.13×10^2 **B** ◯ 1.32×10^2 **C** ◯ 1.55×10^2 **D** ◯ 1.81×10^2
E ◯ 2.12×10^2 **F** ◯ 2.48×10^2 **G** ◯ 2.90×10^2 **H** ◯ 3.40×10^2

 Licensed under GNU General Public License



The vertical component of the force F is half the weight: $F_y = \frac{mg}{2} = $

$= \frac{27.0 \cdot 9.81}{2} = 132\,N$

The weight of a uniform beam is split equally (i.e. 50-50) between the two endpoints of the beam (in the vertical direction).