Unitarity
and
Bounds on the scale of Fermion Mass Generation
in
Deconstructed Higgsless Models
What is the scale of fermion mass generation?

- Is it the same as the scale of EW gauge boson mass generation?

Can we find an upper bound on this scale?

- Yes – PRL 59, 2405 (1987)
  Appelquist and Chanowitz did this for the SM without a Higgs.

- Unitarity breaks down in the process $t\bar{t} \rightarrow W^+_L W^-_L$ if new physics does not appear before $\Lambda_{AC}$.
Is the AC bound truly independent?

  
  Won't the fields that unitarize WW scattering also unitarize
  
  \[ t\bar{t} \rightarrow W^+_L W^-_L \] ?

• In the SM, the Higgs unitarizes both.

• In Higgsless models, distinct fields unitarize
  
  \[ t\bar{t} \rightarrow W^+_L W^-_L \] and WW scattering.
Is the $2\rightarrow m$ process stronger?

  Maltoni, Niczyporuk, and Willenbrock noted that the $2\rightarrow m$ process can sometimes give a stronger bound.

- **PRD 71**, 093009 (2005)
  Dicus and He showed that for the top quark, the $2\rightarrow 2$ process was still the strongest.
How is this scale modified in Higgsless models?

• **PRD 75, 073018 (2007)**
  Chivukula, Christensen, Coleppa and Simmons showed that
  $t\bar{t} \rightarrow W_L^+ W_L^-$ is unitarized by a
  set of fields distinct from those which unitarize WW scattering.

• The scale where unitarity breaks down is a function of the mass of
  the $1^{\text{st}}$ KK mode of the fermions
  and is independent of the mass of the $1^{\text{st}}$ KK mode of the gauge
  bosons.
2-Site Model: (Higgsless SM)
2-Site Model: Gauge

\[ W_0 = \begin{pmatrix} \frac{1}{2} W_0^0 & \frac{1}{\sqrt{2}} W_0^+ \\ \frac{1}{\sqrt{2}} W_0^- & -\frac{1}{2} W_0^0 \end{pmatrix} \]

\[ W_1 = \begin{pmatrix} \frac{1}{2} W_1^0 & 0 \\ 0 & -\frac{1}{2} W_1^0 \end{pmatrix} \]

\[ \mathcal{L}_{F^2} = -\frac{1}{2} \text{Tr} \left[ F_0^2 + F_1^2 \right] \]

where

\[ F_0^{\mu\nu} = \partial^\mu W_0^\mu - \partial^\nu W_0^\mu + ig [W_0^\mu, W_0^\nu] \]

\[ F_1^{\mu\nu} = \partial^\mu W_1^\mu - \partial^\nu W_1^\mu \]
2-Site Model: Gauge-Goldstone

\[ \Sigma_0 = e^{i \frac{2\pi \pi_0}{f}} \]

\[ \pi_0 = \begin{pmatrix} \frac{1}{2} \pi_0^0 & \frac{1}{\sqrt{2}} \pi_0^+ \\ \frac{1}{\sqrt{2}} \pi_0^- & -\frac{1}{2} \pi_0^0 \end{pmatrix} \]

\[ \mathcal{L}_{D\Sigma} = \frac{f^2}{2} \text{Tr} \left[ (D_\mu \Sigma_0)^\dagger D^\mu \Sigma_0 \right] \]

where

\[ D_\mu \Sigma_0 = \partial_\mu \Sigma_0 + igW_0 \Sigma_0 - ig' \Sigma_0 W_1 \]
2-Site Model: Gauge Masses

\[ M^2_{\pm} = \frac{f^2}{4} \left( g^2 \right) \]

\[ M^2_W = \frac{g^2 f^2}{4} \]

\[ v_W = \{1\} \]

\[ \mathcal{L}_{WW} = \frac{f^2}{2} \text{Tr} \left[ (D_\mu I)^\dagger D^\mu I \right] \]

\[ M^2_n = \frac{f^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \]

\[ M^2_\gamma = 0 \quad v_\gamma = e \left\{ \frac{1}{g}, \frac{1}{g'} \right\} \]

\[ M^2_Z = \frac{(g^2 + g'^2) f^2}{4} \]

\[ v_Z = e \left\{ \frac{1}{g'}, -\frac{1}{g} \right\} \]
2-Site Model: Fermion-Gauge

\[ \mathcal{L}_{D\psi} = \bar{\psi}_L \partial \psi_L + \bar{\psi}_R \partial \psi_R \]

where

\[ D_\mu \psi_L = \partial_\mu \psi_L + i g W_0 \psi_L + i g' Y_{0f} W_1 \psi_L \]

\[ D_\mu \psi_R = \partial_\mu \psi_R + i g' Y_{1f} W_1 \psi_R \]

\[ Y_{0Q} = 1/3 \quad Y_{1u} = 4/3 \]

\[ Y_{1d} = -2/3 \]

\[ Y_{0L} = -1 \quad Y_{1e} = -2 \]
2-Site Model: Fermion-Goldstone

\[ \mathcal{L} = -M_F \epsilon_R \bar{\psi}_L \Sigma_0 \psi_{R1} \]

\[ m_f = M_F \epsilon_R \]

\[ \nu_L = \{1\} \]

\[ \nu_R = \{1\} \]
AC Bound 1

Helicities and colors are summed over for a stronger bound:

\[ |\psi\rangle = \frac{1}{\sqrt{6}} \left( |\bar{t}_1 t_1\rangle + |\bar{t}_2 t_2\rangle + |\bar{t}_3 t_3\rangle - |\bar{t}_1 t_1\rangle - |\bar{t}_2 t_2\rangle - |\bar{t}_3 t_3\rangle \right) \]
AC Bound 2

Leading order expressions in $M_W^2, m_t^2/s$ are used:

\[
\epsilon_{W_L}^\mu \sim \frac{k_{W_L}^{\mu}}{M_W}
\]

\[
\bar{v}_+ (k_1 - k_2) (g_L P_L + g_R P_R) u_+ \sim m_t \sqrt{s} \cos (g_L + g_R)
\]

\[
\bar{v}_+ k_2 (\not{v}_1 - \not{k}_1) k_1 g_L P_L u_+ \sim \frac{m_t t \sqrt{s}}{2} (1 + \cos \theta) g_L
\]
The contribution from the gauge bosons cancels with part of the contribution from the b quark:

\[ 2g_{tt\gamma}g_{\gamma WW} + g_{Lt\tau Z}g_{ZW} + g_{Rtt\tau Z}g_{ZW} - g^2_{Lt\tau bW} = 0 \]
\[ M \approx \frac{\sqrt{6s} m_t}{2M_W^2} g_{LtbW}^2 = \frac{\sqrt{6s} m_t}{v^2} \]
Only the 4 point vertex contributes at order $\sqrt{s}$.

\[ M \simeq \sqrt{6s} \ g_{tt\pi\pi} = \frac{\sqrt{6s} \ m_t}{v^2} \]
The J=0 partial wave amplitude is calculated.
The real part must be less than $\frac{1}{2}$ for unitarity.
This gives the Appelquist-Chanowitz bound.

\[
a_0 = \frac{1}{32\pi} \int_{-1}^{1} dc\cos\theta \ M < \frac{1}{2}
\]

\[
a_0 \sim \frac{m_t \sqrt{6}s}{16\pi v^2}
\]

\[
\sqrt{s} \lessapprox \frac{8\pi v^2}{m_t \sqrt{6}} \sim 3.5\text{TeV}
\]

• We know that unitarity breaks down in the channel
  \[ W_L^+ W_L^- \rightarrow W_L^+ W_L^- \]

• Some new physics has to appear before \( \sim \) 1 TeV to unitarize WW scattering.

• Won't the fields that unitarize \( W_L^+ W_L^- \rightarrow W_L^+ W_L^- \)
  also unitarize \( t_+ \bar{t}_+ \rightarrow W_L^+ W_L^- \)?

• Consider the Higgs:
  
  It unitarizes both processes.
AC Bound: WW scattering

- This process becomes nonunitary at $\sqrt{s} \sim 1 TeV$.
- New scalar fields could help unitarize this process.
- New vector fields could help unitarize this process.
- New fermions could not help unitarize this process.
AC Bound: $J=0 \quad t_+\bar{t}_+ \rightarrow W_L^+W_L^-$

- This process becomes nonunitary at $\sqrt{s} \sim 3.5 \text{TeV}$.
- New scalar fields could help unitarize this process.
- New fermions could help unitarize this process.
- New vector fields could not help unitarize this process.
  (Vector fields in the S channel do not contribute to $J=0$.)
AC Bound: The Higgs

A scalar field has the potential of unitarizing both $WW$ scattering and ...
But, in a Higgsless model, there are no scalars.

A viable Higgsless model must:

- Unitarize $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ with gauge bosons.
- Unitarize $t_+ \bar{t}_+ \rightarrow W_L^+ W_L^-$ with fermions.
Higgsless Unitarity of WW Scattering

- WW scattering is unitarized by exchange of an infinite tower of Kaluza-Klein modes of the Z boson.
Higgsless Unitarity of \( t_+\bar{t}_+ \rightarrow W_L^+W_L^- \)

\[ \begin{array}{c}
  t_+ \\
  \downarrow \\
  b, B_k \\
  \downarrow \\
  \bar{t}_+ \\
\end{array} \]

- \( t_+\bar{t}_+ \rightarrow W_L^+W_L^- \) in the J=0 channel, is unitarized by the exchange of an infinite tower of Kaluza-Klein modes of the bottom quark.

They noticed that $2 \rightarrow m$ may give a stronger bound than $2 \rightarrow 2$.

They estimated $g_{tt\pi m}$.
$2 \to m : \text{MNW}$

\[ I_m = \int \frac{d^3 k_1 \cdots d^3 k_m}{2E_1 \cdots 2E_m} \delta^{(4)}(P - k_1 - \cdots - k_m) \]

\[ \sim s^{m - 2} \]

• They estimated the phase space.
Putting these together, they found that unitarity was bounded by a scale that approached $v$ as the number of final states approached $\infty$. 

$$2 \rightarrow m : \text{MNW}$$

$$\sigma \sim \left( \frac{m_t}{vm} \right)^2 s^{m-2} \lesssim \frac{4\pi}{s}$$

$$\sqrt{s} \lesssim \left( \frac{vm}{m_t} \right)^{\frac{1}{m-1}} \quad m \rightarrow \infty \quad v$$
$2 \rightarrow m : \text{DH}$

$\sqrt{s} \geq m M_W$

- **PRD 71, 093009 (2005):** Dicus and He
- Shouldn't there be at least enough energy to produce the final state particles?
They carefully calculated the phase space and found the important factors \((m-1)!(m-2)!\) in the denominator.
They found that the unitarity bound actually approaches \( mM_w \) as the number of final states approaches \( \infty \).

For some particles, the bound does become stronger with increased number of final states.

However, they found that for the top quark, the \( 2 \to 2 \) process still gives the strongest bound.
$2 \rightarrow m : \text{DH}$
n(+2) Site Model: Introduction
n(+2) Site Model: Gauge

\[ \mathcal{L}_{F^2} = -\frac{1}{2} \text{Tr} \left[ \Sigma_j F_j^2 \right] \]

where

\[ F_{j}^{\mu\nu} = \partial^{\mu} W_{j}^{\nu} - \partial^{\nu} W_{j}^{\mu} + ig \left[ W_{j}^{\mu}, W_{j}^{\nu} \right] \]

\[ F_{n+1}^{\mu\nu} = \partial^{\mu} W_{n+1}^{\nu} - \partial^{\nu} W_{n+1}^{\mu} \]

\[ W_{j} = \begin{pmatrix} \frac{1}{2} W_{j}^{0} & \frac{1}{\sqrt{2}} W_{j}^{+} \\ \frac{1}{\sqrt{2}} W_{j}^{-} & -\frac{1}{2} W_{j}^{0} \end{pmatrix} \]

\[ W_{n+1} = \begin{pmatrix} \frac{1}{2} W_{n+1}^{0} & 0 \\ 0 & -\frac{1}{2} W_{n+1}^{0} \end{pmatrix} \]
n(+2) Site Model: Gauge-Goldstone

\[ \mathcal{L}_{D\Sigma} = \frac{f^2}{2} \text{Tr} \left[ (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j \right] \]

where

\[ D_\mu \Sigma_j = \partial_\mu \Sigma_j + ig_j W_j \Sigma_j - ig_{j+1} \Sigma_j W_{j+1} \]

\[ \Sigma_j = e^{i \frac{2\pi j}{f}} \]

\[ \pi_j = \begin{pmatrix} \frac{1}{2} \pi^0_j & \frac{1}{\sqrt{2}} \pi^+_j \\ \frac{1}{\sqrt{2}} \pi^-_j & -\frac{1}{2} \pi^0_j \end{pmatrix} \]
n(+2) Site Model: Gauge Bosons

\[ M_{Z0} = \frac{g f}{2 c \sqrt{n + 1}} \]

\[ v_{Z0}^0 = c \]

\[ v_{Z0}^j = \frac{c(n + 1) - j/c}{n + 1} x \]

\[ v_{Z0}^{n+1} = -s \]

\[ M_{W0} = \frac{g f}{2 \sqrt{n + 1}} \]

\[ v_{W0}^0 = 1 \]

\[ v_{W0}^j = \frac{n - j + 1}{n + 1} \]

\[ \mathcal{L}_{WW} = \frac{f^2}{2} \text{Tr} \left[ (D_\mu I) \dagger D^\mu I \right] \]

\[ M_n^2 = \frac{g^2 f^2}{4} \begin{pmatrix}
  x^2 & -x & 0 & 0 & \cdots & 0 & 0 \\
  -x & 2 & -1 & 0 & \cdots & 0 & 0 \\
  0 & -1 & 2 & -1 & \cdots & 0 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & -1 & 2 & -xt \\
  0 & 0 & 0 & \cdots & 0 & -xt & x^2 l^2 
\end{pmatrix} \]
n(+2) Site Model: Fermion-Gauge

\[ Y_{jQ} = 1/3 \quad Y_{n+1,u} = 4/3 \]
\[ Y_{n+1,d} = -2/3 \]
\[ Y_{jL} = -1 \quad Y_{n+1,e} = -2 \]

\[ \mathcal{L}_{D \psi} = \sum_j \bar{\psi}_j D \psi_j \]

where

\[ D_\mu \psi_j = \partial_\mu \psi_j + ig W_j \psi_j + ig' Y_{j1} W_1 \psi_j \]

\[ D_\mu \psi_{R,n+1} = \partial_\mu \psi_{R,n+1} + ig' Y_{n+1,f} W_1 \psi_{R,n+1} \]
n(+2) Site Model: Fermion-Goldstone

\[ \mathcal{L}_{\psi} = -M_F \left[ \epsilon_L \bar{\psi}_{L0} \Sigma_0 \psi_{R1} - \sum_j \bar{\psi}_{Lj} \psi_{Rj} + \sum_j \bar{\psi}_{Lj} \Sigma_j \psi_{R,j+1} + \bar{\psi}_{Ln} \epsilon_R \Sigma_n \psi_{R,n+1} + \text{H.c.} \right] \]

\[ M_{F_0} = M_F \epsilon_L \epsilon_{R_f} \]

\[ u_0^{LF_0} = 1 \]

\[ u_j^{LF_0} = \epsilon_L \]

\[ u_j^{RF_0} = \epsilon_{R_f} \]

\[ u_{n+1}^{RF_0} = 1 \]
n(+2) Site Model: $S$

*Alterations of $g_{\nu\nu}$ can be parametrized by $S$.
*S can be calculated in the $n(+2)$ site model and set to zero.

$$g_{\nu\nu} = \frac{e}{s_M} \left( 1 + \frac{\alpha}{4s_M^2} S \right)$$

$$g_{\nu\nu} = \frac{e}{s_M} \left( 1 + \frac{n(n + 2)}{6(n + 1)} x^2 - \frac{n}{2} \epsilon_L^2 \right)$$

$$S = 0 \implies \epsilon_L^2 = \frac{n + 2}{3(n + 1)} x^2$$
n(+2) Site Model: Goldstone Bosons

- The Goldstone bosons are determined by their mixing with the gauge bosons that eat them.
- The Goldstone bosons eaten by the W and Z are particularly simple.

\[ \mathcal{L}_{D\Sigma} = \frac{f^2}{4} \text{Tr} \left[ \sum_j (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j \right] \]

\[ \mathcal{L}_{\pi W} = -i \frac{\tilde{g} f}{2} \left[ \{ \partial_\mu \pi_0, x W_0^\mu - W_1^\mu \} \right. \]

\[ + \sum_{j=1}^{n-1} \left\{ \partial_\mu \pi_j, W_j^\mu - W_{j+1}^\mu \right\} \]

\[ + \left\{ \partial_\mu \pi_n, W_n^\mu - x t W_{n+1}^\mu \right\} \]

\[ \nu_{\pi_0}^{[1]} = \frac{1}{\sqrt{n + 1}} = \nu_{\pi_0}^{[1]} \]
n(+2) Site Model: Couplings

\[ \mathcal{L}_{\psi_\Sigma} = -M_F \left[ \epsilon_L \bar{\psi}_L \Sigma_0 \psi_{R1} - \sum_j \bar{\psi}_{Lj} \psi_{Rj} + \sum_j \bar{\psi}_{Lj} \Sigma_j \psi_{R,j+1} + \bar{\psi}_{Ln} \epsilon_R \Sigma_n \psi_{R,n+1} + \text{H.c.} \right] \]
n(+2) Site Model: Couplings

\[ \mathcal{L}_{\psi_S} = -M_F \left[ \epsilon_L \bar{\psi}_{L0} \Sigma_0 \psi_{R1} - \sum_j \bar{\psi}_{Lj} \psi_{Rj} \right. \]

\[ \left. + \sum_j \bar{\psi}_{Lj} \Sigma_j \psi_{R,j+1} + \bar{\psi}_{Ln} \epsilon_R \Sigma_n \psi_{R,n+1} + \text{H.c.} \right] \]

\[ g_{\pi^+\pi^-} = \frac{M_F}{f^2} \left[ \epsilon_L v^0_{Li} v^1_{Ri} (v^0_{\pi})^2 + \sum_i v^i_{Li} v^{i+1}_{Ri} (v^i_{\pi})^2 \right. \]

\[ \left. + \epsilon_R v^n_{Li} v^{n+1}_{Ri} (v^n_{\pi})^2 \right] \]

\[ = \frac{m_t}{(n + 1)u^2}, \]
n(+2) Site Model: Calculation

- The 4 point diagram grows like $\sqrt{s}$ for all energies.
- The T channel diagrams grow like $\sqrt{s}$ up to $M_{F_k}$.
- It is the $F_k$ that unitarize this process and not the $W_k$!

\[ \mathcal{M} = \sqrt{6s} \left( g_{tt\pi^+\pi^-} - \sum_k \frac{M_{F_k} g_{LtF_k\pi} g_{RtF_k\pi}}{t - M_{F_k}^2} \right) \]

\[ a_0 = \frac{1}{32\pi} \int_{-1}^{1} d\cos\theta \mathcal{M} \]

\[ = \frac{\sqrt{6}}{16\pi} \left[ g_{tt\pi^+\pi^-}\sqrt{s} + \sum_k g_{LtF_k\pi} g_{RtF_k\pi} g \left( \frac{\sqrt{s}}{M_{F_k}} \right) \right] \]

\[ g(x) = \frac{1}{x} \ln(1 + x^2) \]
n(+2) Site Model: Unitarity Bound

\[ \sqrt{s} \text{ (TeV)} \]

\[ M_{F1} \text{ (TeV)} \]

\[ \infty = n \]
For $M_{F_1} \ll 4.5 \text{ TeV}$, the bound is determined by the 4 point vertex. 
In that limit, the bound is just a multiple of the AC bound. 
The bound disappears in the continuum limit.
n(+2) Site Model: $n \rightarrow \infty$

- The edge can be determined in the $n \rightarrow \infty$ limit where the 4 point vertex disappears.
- The T channel is dominated by the first KK mode.

$$\lim_{n \rightarrow \infty} a_0 = \frac{2 \sqrt{6} M_{F_1} m_t}{\pi^4 v^2} \sum_k \frac{(-1)^{k+1}}{(2k-1)^2} g\left(\frac{\sqrt{s}}{(2k-1)M_{F_1}}\right)$$

$$\lim_{n \rightarrow \infty} a_0(k = 1) \approx \frac{2 \sqrt{6} M_{F_1} m_t}{\pi^4 v^2} g\left(\frac{\sqrt{s}}{M_{F_1}}\right)$$

$$M_{F_1} \lesssim \frac{\pi^4 v^2}{2 \sqrt{6} m_t \ln(5)} \sim 4.25 \text{ TeV}$$
In Higgsless models:

- The process $t_+ \bar{t}_+ \rightarrow W_L^+ W_L^-$ is unitarized by $B_k$ while $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ is unitarized by $Z_k$.

- The bound on the scale of fermion mass generation is independent of the scale of gauge boson mass generation.

- Even for a small number of new 'sites', the scale where new physics responsible for the mass generation of the fermions appears can be significantly altered and weakened by the presence of mixing between the fields of the different 'sites'.

Summary
Appendix
$2 \to m : n(+2) \text{ site}$

- These vertices are further suppressed by $1/n^m$.
- These vertices disappear as $n \to \infty$.

$$g_{tt\pi^m} = \frac{2^m M_F}{(\sqrt{2})^m m! f m} \left[ \epsilon_L v_L^0 v_R^1 (v_{\pi}^0)^m + \sum_j v_{L t}^j v_{R t}^{j+1} (v_{\pi}^j)^m + \epsilon_{R t} v_{L t}^n v_{R t}^{n+1} (v_{\pi}^n)^m \right]$$

$$g_{tt\pi^m} = \frac{(\sqrt{2})^m m_t}{m!(n + 1)^{m-1} v^m}$$