Clocking hadronization in relativistic heavy ion collisions with balance functions

Steffen A. Bass, Pawel Danielewicz and Scott Pratt
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory,
Michigan State University, East Lansing, MI 48824 USA
(June 22, 2005)

A novel state of matter has been hypothesized to exist during the early stage of relativistic heavy ion collisions, with normal hadrons not appearing until several fm/c after the start of the reaction. To test this hypothesis, correlations between charges and their associated anticharges are evaluated with the use of balance functions. It is shown that late-stage hadronization is characterized by tightly correlated charge-anticharge pairs when measured as a function of relative rapidity.

Relativistic heavy ion collisions produce mesoscopic regions of enormous energy density, perhaps surpassing 3 GeV/fm^3 in Pb collisions at the CERN SPS [1,2] with even higher energy densities expected at RHIC. At such energies hadronic degrees of freedom should be replaced by quark-gluon degrees of freedom. Several experimental measurements have been proposed as signals to the quark-gluon plasma [3]. Among these signals is an expected enhancement in strange-quark production which should take place 5-10 fm/c into the collision when the local temperature has dropped to near 160 MeV, but the system is still far from freeze-out. Strangeness enhancement has indeed been observed in heavy ion collisions [4], but alternative hadronic explanations have also been put forward assuming early-stage hadronization with medium modifications, referred to as color ropes [5,6] or baryon junctions [7]. In this paper the use of balance functions is proposed as a means to determine whether quark production occurred at early times, $\tau < 1$ fm/c, or according to a late-stage hadronization scenario, see e.g. [8,9].

Late-stage production of quarks could be attributed to three mechanisms: formation of hadrons from gluons, conversion of the non-perturbative vacuum energy into particles, or hadronization of a quark gas at constant temperature. Hadronization of a quark gas should approximately conserve the net number of particles due to the constraint of entropy conservation. Since hadrons are formed of two or more quarks, creation of quark-antiquark pairs should accompany hadronization. All three mechanisms for late-stage quark production involve a change in the degrees of freedom. Therefore, any signal that pinpoints the time where quarks first appear in a collision would provide valuable insight into understanding whether a novel state of matter has been formed and persisted for a substantial time. The fact that the hadronic phase has a higher concentration of charges than the QGP phase at the same entropy has been discussed in the context of charge fluctuations in [10].

The link between balance functions and the time at which quarks are created has a simple physical explanation. Charge-anticharge pairs are created at the same location in space-time, and are correlated in rapidity due to the strong collective expansion inherent to a relativistic heavy ion collision. Pairs created earlier can separate further in rapidity due to the higher initial temperature and due to the diffusive interactions with other particles. The balance function, which describes the momentum of the accompanying antiparticle, quantifies this correlation.

The balance functions employed here are similar to observables used to investigate hadronization in jets produced in $p\bar{p}$ or e^+e^- collisions [11,12]. The balance function describes the conditional probability that a particle in the bin p_1 will be accompanied by a particle of opposite charge in the bin p_2. We define the balance function,

$$B(p_2|p_1) \equiv \frac{1}{2}\left\{\rho(b,p_2|a,p_1) - \rho(b,p_2|b,p_1) + \rho(a,p_2|b,p_1) - \rho(a,p_2|a,p_1)\right\}, \tag{1}$$

where $\rho(b,p_2|a,p_1)$ is the conditional probability of observing a particle of type b in bin p_2 given the existence of a particle of type a in bin p_1. The label a might refer to all negative kaons with b referring to all positive kaons, or a might refer to all hadrons with a strange quark while b refers to all hadrons with an antistrange quark. The conditional probability $\rho(b,p_2|a,p_1)$ is generated by first counting the number $N(b,p_2|a,p_1)$ of pairs that satisfy both criteria and dividing by the number $N(a,p_1)$ of particles of type a that satisfy the first criteria.

$$\rho(b,p_2,a,p_1) = \frac{N(b,p_2|a,p_1)}{N(a,p_1)}. \tag{2}$$

Both sums run over all events, though pairs only involve particles from the same event.

An example of binning might be that p_1 refers to a measurement anywhere in the detector, while p_2 refers to the relative rapidity $|y_b - y_a|$. Then the balance function would be a function of Δy only, and would represent the probability that the balancing charges were separated by Δy (in our formalism we include a division by Δy to express $B(\Delta y)$ as a density).

The balance function is normalized to unity if a/b refer to all particles with a positive/negative globally conserved charge.

$$\sum_{p_2} B(p_2|p_1) = \frac{1}{2}\left\{M_b - (M_b - 1) + M_a - (M_a - 1)\right\} = 1, \tag{3}$$

where M_a and M_b are the average multiplicities of the a and b particles. The normalization derives from the fact that for every extra positive charge there exists one extra negative charge. If the acceptance measures only a fraction of the charge, e.g. only kaons are measured and the strangeness in hyperons is excluded, the balance function would sum to that fraction. Balance functions can exploit any conserved charge: electric charge, strangeness, baryon number or charm. The first two terms in Eq. (1) constitute the balance functions defined in several analyses of $e^+e^- \rightarrow$ jets. By adding the last two terms the normalization properties are retained even for the case where there is a non-zero net charge, $M_a - M_b \neq 0$.

If many charges are present in the event, the balance function represents the subtraction of two large numbers. However, large multiplicities also imply a large number of pairs from which to calculate the balance function. Since the number of uncorrelated pairs rises as the square of the multiplicity M, the statistical error in calculating the numerators of the conditional probabilities, which rises as the square root of the number of pairs, increases linearly with M. Since the denominator also rises linearly with M, the statistical error in the balance function is independent of multiplicity and is principally determined by the number of events:

$$\sigma_B \propto \frac{1}{\sqrt{N_{\rm ev}}}. \tag{4}$$

Thus, the baryon-antibaryon balance function which might involve a few dozen antibaryons would require the same number of events as the electric-charge balance function which might be constructed from a thousand particles. Typically, 10^5 events are required to determine a balance function with statistical fluctuations at the level of 10^{-2}.

Balance functions probe the dynamics of charge-anticharge pairs by quantifying the degree to which the charges are correlated in momentum space given the constraint of being created at the same space-time point in a system exhibiting strong position-momentum correlations such as a relativistic collision where source velocities might span several units of rapidity. In a globally equilibrated system with no collective flow, there would exist no correlation between the balancing charges, and the numerator in Eq. (2) would factorize. The width of the balance function would then correspond to the extent of single-particle emission in momentum space.

To illustrate the way in which balance functions quantify the charge-anticharge correlations, we consider a Bjorken boost-invariant parameterization [13] of a source expanding along the z axis with a collective velocity proportional to the position, $v_{\rm coll} = z/t$. All intrinsic variables, such as density or temperature, depend only on the proper time $\tau = (t^2 - z^2)^{1/2}$. We first consider only direct production of hadrons, as the possibility of hadrons coalescing from quarks is discussed later in the paper. Particles and antiparticles of mass m are generated in pairs at the same point in space-time following a local thermal distribution, and the relative rapidities are used to generate balance functions. The characteristic width of the balance function is determined by the ratio of the temperature to the mass. Non-relativistically, $\sigma_y = (2T/m)^{1/2}$, and heavier particles are characterized by narrower balance functions. For particles with masses much less than the temperature, the balance functions become independent of the temperature.

Figure 1 displays balance functions assuming a Bjorken parameterization of an expanding pion gas and an expanding proton gas, for two temperatures, 225 MeV and 165 MeV. Clearly, the balance functions of the more massive particles are sensitive to the temperature. This suggests that the strangeness and baryon balance functions should provide more insight than the electric-charge balance function which would be largely dominated by pions.

Balance functions in heavy ion collisions should be compared to those from pp collisions at the same $\sqrt{s}$ where hadronization is nearly instantaneous. Charged-pion balances measured in e^+e^- collisions as a function of the rapidity defined along the jet axis have been reasonably explained by the string hadronization dynamics of the Lund model [14], e.g. as implemented in PYTHIA [15]. Thermally generated balance functions are compared to predictions of PYTHIA for pp collisions at $\sqrt{s} = 200$ GeV in Fig. 1. The PYTHIA balance functions tend to be broader than those that are thermally generated, especially for the more massive protons and kaons. Assuming that experimental balance functions in pp collisions would be well described by similar string dynamics, Fig. 1 suggests that narrower balance functions might indeed point to thermal production at a lower temperature and thus at later times in the evolution of the heavy ion reaction.

Rescattering and annihilation should also affect balance functions. Rescattering may be qualitatively understood by considering the diffusion equation in Bjorken coordinates τ and $\eta \equiv \tanh^{-1}(z/t)$, where η plays the role of the position in the z direction and also equals the collective rapidity of the local matter. Rather than considering the diffusion constant $D = v_t/(n\sigma)$ as a constant, it is more physical to incorporate the fact that the density n falls inversely with τ and to consider $\beta \equiv v_t/(n\tau\sigma)$ as a constant where v_t is the thermal velocity and σ is a characteristic cross section. The diffusion equation then becomes

$$\frac{\partial}{\partial\tau} f(\tau,\eta) = -\frac{\beta}{\tau}\frac{\partial^2}{\partial\eta^2} f(\tau,\eta). \tag{5}$$

Here, f is the probability of observing a particle at position η at time τ. With the initial condition of $\eta = 0$ at τ_0, the solution to the diffusion equation is a Gaussian with variance $\sigma_\eta^2 = 2\beta \ln(\tau/\tau_0)$. This illustrates that collisions broaden the balance function by diffusing the charge in the effective spatial coordinate η. However, in the limit of zero mean free path, the diffusion constant tends to zero and the particles do not then diffuse.

The overall width of the balance function in relative rapidity is a combination of the thermal rapidity spread $\sigma_{\rm therm}$ and the effect of diffusion in η of both particles:

$$\sigma_y^2 = \sigma_{\rm therm}^2 + 4\beta \ln(\tau/\tau_0). \tag{6}$$

Due to cooling, the width $\sigma_{\rm therm}$ falls with time which provides a competition between diffusion which stretches the balance function, and cooling which narrows it. If the production occurs at early times, then $\ln(\tau/\tau_0)$ is large and the effect of collisions is to significantly broaden the balance function.

Some hadrons will contain coalesced quarks that were created at early times. The thermal contribution to σ_y described in Eq. (6) should be unaffected by the past history of the constituent quarks. However, the diffusive contribution might significantly depend on the fact that the charge moved as a free quark rather than as a hadron during it's early history. Balance functions constructed from hadrons can thus provide meaningful information regarding the creation and mobility of the constituent quarks.

To quantitatively illustrate the effect of rescattering, we model a pair of particles produced at an initial proper time τ_0 that collide $N_{\rm coll}$ times before disassociating at a final time τ_f. Each collision is assumed to completely reorient the particle with the local collective velocity. The collision times are chosen randomly such that the number of collisions as a function of $\ln(\tau)$ is uniform. The temperature is chosen to vary linearly with the proper time, cooling from 225 MeV at $\tau = 1$ fm/c to 120 MeV at $\tau = 15$ fm/c. Figure 2 shows the K_+K_- balance function with $N_{\rm coll} = 0$ and $N_{\rm coll} = 10$ assuming kaons are created at $\tau = 1$ fm/c and cease to collide at $\tau_f = 15$fm/c. In this case collisions clearly broaden the balance function.

Annihilations should also broaden the balance function. Annihilation forms new correlated pairs with the surviving partners of the annihilated particles, which tend to be less correlated than the original pairs. Annihilation combined with an equal amount of creation does not affect the balance function since the relative rapidities of formed and annihilated pairs should be identical. Figure 2 illustrates the effects of annihilation by considering the same case described above, but with the additional assumption that half the particles disappear due to annihilation. In hadronic models of heavy ion collisions, the number of both antibaryons and strange particles tend to decrease with time due to cooling, which should result in broadened balance functions.

Figure 3 displays the effect of collisions on balance functions for pions, kaons and protons, by considering the mean relative rapidity as a function of the number of collisions. For production at early times when the collective velocity gradient is high ($dv_{\rm coll}/dz = 1/\tau$), collisions broaden the balance function. However, for very large numbers of collisions, the charge does not diffuse and the balance functions are narrowed due to the cooling. One would expect particles to undergo 10-20 collisions if created at $\tau = 1$ fm/c, although the effective number of completely randomizing collisions might be closer to a half dozen. If created at $\tau = 9$ fm/c when the temperature is 165 MeV, the effective number of completely randomizing collisions might be two or three. Figure 3 suggests that the signal for late-stage quark production is significantly magnified by rescattering. Due to collisions, even charged-pion balance functions become strongly sensitive to the creation time.

The simple calculations presented here sidestep two issues: correlations from decays such as $\phi \to K^+K^-$, and experimental acceptance problems. Both problems can be addressed by modeling constrained by the multitude of other observables measured in a heavy ion collision. Although some open questions remain, it seems clear that the canonical picture of a heavy-ion reaction,quark-gluon plasma formation followed by late-stage hadronization, should have a clear signature in the balance functions. Compared to pp collisions, one expects the peak in the balance function in nucleus-nucleus collisions to be narrower near $\Delta y = 0$ due to the contribution of late-stage production of quark pairs, while the tails of balance function should become broader reflecting the extra diffusion of charge in the early stages of the collision. Finally, we remark that we have barely explored the possibilities of balance functions. The rich nature of the binnings $(p_2|p_1)$ should provide a powerful means for resolving many of the issue regarding creation and diffusion of quarks and hadrons in relativistic heavy ion collisions.

ACKNOWLEDGMENTS

We are grateful to T. Sjöstrand for providing valuable references. This work was supported by the National Science Foundation, Grants No. PHY-00-70818 and PHY-96-0527.

[1] T. Alber et al. Phys. Rev. Lett. **74** (1995) 1303.
[2] M .Aggarwal et al. Nucl. Phys. **A610** (1996) 200c.
[3] J. Harris and B. Müller, Ann. Rev. Nucl. Part. Sci. **46** (1996) 71.
S.A. Bass, M. Gyulassy, H. Stöcker and W. Greiner, J. Phys. **G25** (1999) R1.
[4] E. Andersen et al. (WA97 collaboration), Phys. Lett **B433** (1998) 209.
R. Lietava et al. (WA97 collaboration), Journal of Physics **G25** (1999) 181.
R. Caliandro et al. (WA97 collaboration), Journal of Physics **G25** (1999) 171.
S. Margetis et al. (NA49 collaboration), Journal of Physics **G25** (1999) 189.
F. Gabler et al. (NA49 collaboration), Journal of Physics **G25** (1999) 199.
D. Evans et al. (WA85 and WA94 collaborations), Journal of Physics **G25** (1999) 209.
[5] T. S. Biro, H. B. Nielsen, J. Knoll, Nucl. Phys. **B245** (1984) 449.
J. Knoll, Z. Phys. **C38** (1988) 187.
[6] H. Sorge, M. Berenguer, H. Stöcker, W. Greiner, Phys. Lett. **B289** (1992) 6.
[7] S. E. Vance and M. Gyulassy, Phys. Rev. Lett. **83** (1999) 1735.
[8] J. Rafelski, B. Müller, Phys. Rev. Lett. **48** (1982) 1066; Erratum-ibid.**56** (1986) 2334.
[9] P. Koch, B. Müller, J. Rafelski, Phys. Rep. **142** (1986) 167.
[10] S. Jeon and V. Koch, LANL report hep-ph/0003168;
M. Asakawa, U. Heinz and B. Müller, LANL report hep-ph/0003169.
[11] D. Drijard et al., Nucl. Phys. **B155** (1979) 269.
D. Drijard et al., Nucl. Phys. **B166** (1980) 233.
I.V. Ajinenko et al., Z. Phys. **C43** (1989) 37.
[12] R. Brandelik et al., Phys. Lett. **B100** (1981) 357.
M. Althoff et al., Z. Phys. **C17** (1983) 5.
H. Aihara et al., Phys. Rev. Lett. **53** (1984) 2199.
H. Aihara et al., Phys. Rev. Lett. **57** (1986) 3140.
P.D. Acton et al., Phys. Lett. **B305** (1993) 415.
[13] J.D. Bjorken, Phys. Rev. D**27** (1983) 140.
[14] B. Anderson et al. Nucl. Phys. **B281** (1987) 289.
[15] H.-U. Bengtsson and T. Sjöstrand, Comp. Phys. Com. **46** (1987) 43.

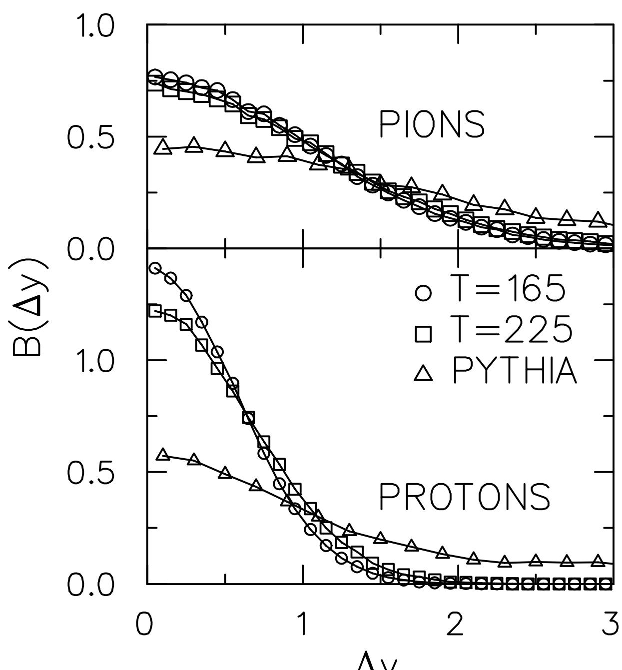

FIG. 1. Balance functions as predicted in a simple Bjorken thermal model are shown for two temperatures, 225 MeV and 165 MeV. Since heavier particles from cooler systems have smaller thermal velocities, they are more strongly correlated in rapidity and result in narrower balance functions. Also shown are balance functions as predicted by PYTHIA where the shape of the balance function is largely determined by string phenomenology.

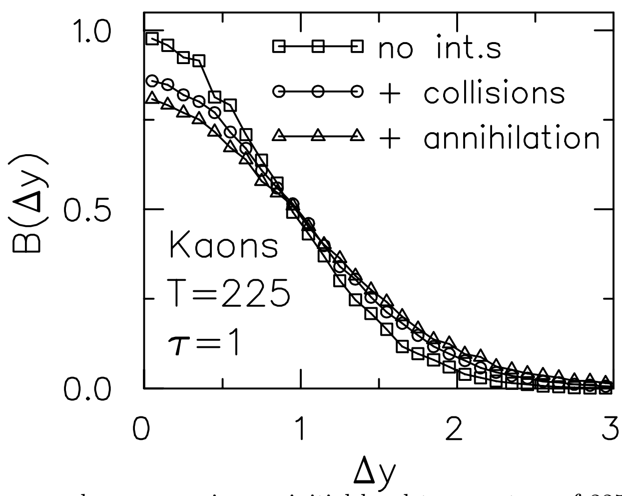

FIG. 2. Kaon balance functions are shown assuming an initial local temperature of 225 MeV and a production time of 1 fm/c. The balance function is broadened by the inclusion of randomizing collisions and annihilation.

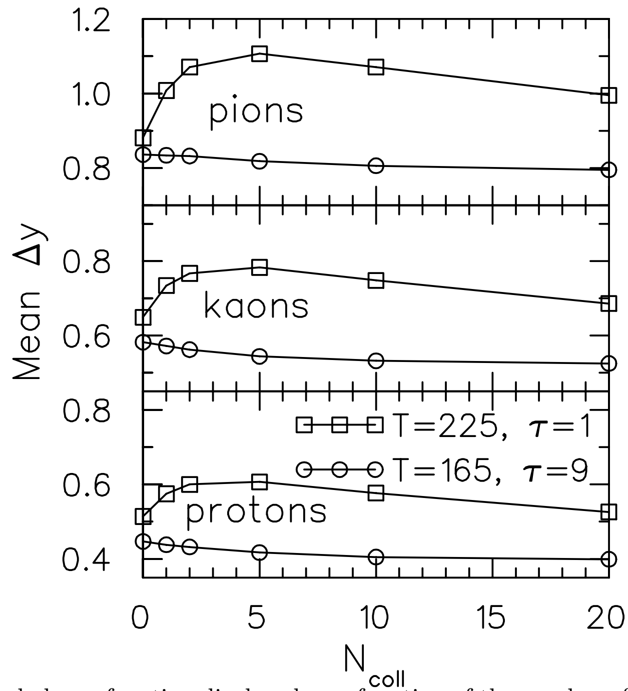

FIG. 3. The mean width of the balance function displayed as a function of the number of collisions, both for the case where particles are created early ($\tau = 1$ fm/c, $T = 225$ MeV) and late ($\tau = 9$ fm/c, $T = 165$ MeV).