Chapter 5
Work and Energy

Forms of Energy
• Mechanical
  • Kinetic, gravitational
• Thermal
  • Microscopic mechanical
• Electromagnetic
• Nuclear

Energy is conserved!

Work
• Relates force to change in energy
  \[ W = F \cdot (x_f - x_i) = F \Delta x \cos \theta \]
• Scalar quantity
• Independent of time

Units of Work and Energy
\[ W = F \cdot x \]
SI unit = Joule
1 J = 1 N\cdot m = 1 kg\cdot m^2/s^2

Work can be positive or negative
• Man does positive work lifting box
• Man does negative work lowering box
• Gravity does positive work when box lowers
• Gravity does negative work when box is raised

Kinetic Energy
\[ KE = \frac{1}{2}mv^2 \]
Same units as work

Remember the Eq. of motion
\[ \frac{v_f^2}{2} - \frac{v_i^2}{2} = a \Delta x \]
Multiply both sides by m,
\[ \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = ma \Delta x \]
\[ KE_f - KE_i = F \Delta x \]
**Example 5.1**
A skater of mass 60 kg has an initial velocity of 12 m/s. He slides on ice where the frictional force is 36 N. How far will the skater slide before he stops?

120 m

**Potential Energy**

If force depends on distance,

\[ \Delta PE = -F \Delta x \]

For gravity (near Earth’s surface)

\[ \Delta PE = mgh \]

**Conservation of Energy**

\[ PE_f + KE_f = PE_i + KE_i \]

\[ \Delta KE = -\Delta PE \]

**Conservative forces:**
- Gravity, electrical, QCD...

**Non-conservative forces:**
- Friction, air resistance...

Non-conservative forces still conserve energy!
Energy just transfers to thermal energy

**Example 5.2**
A diver of mass \( m \) drops from a board 10.0 m above the water surface, as in the Figure. Find his speed 5.00 m above the water surface. Neglect air resistance.

9.9 m/s

**Example 5.3**
A skier slides down the frictionless slope as shown. What is the skier’s speed at the bottom?

28.0 m/s
Example 5.4
A skier slides down the frictionless 40-m high slope as shown, then reaches a flat area with \( \mu_k = 0.2 \). What distance does he slide before stopping?

Example 5.5
Two blocks, A and B (mass A = 50 kg and mass B = 100 kg), are connected by a string as shown. If the blocks begin at rest, what will their speeds be after A has slid a distance \( s = 0.25 \) m? Assume the pulley and incline are frictionless.

Example 5.6
Three identical balls are thrown from the top of a building with the same initial speed. Initially, Ball 1 moves horizontally, Ball 2 moves upward, and Ball 3 moves downward. Neglecting air resistance, which ball has the fastest speed when it hits the ground?
A) Ball 1
B) Ball 2
C) Ball 3
D) All have the same speed.

Example 5.7
Tarzan swings from a vine whose length is 12 m. If Tarzan starts at an angle of 30 degrees with respect to the vertical and has no initial speed, what is his speed at the bottom of the arc?

5.61 m/s

"Energy" conservation

Non-conservative!

Toll Bridges

$2.50

$3.00

$1.00

Still not conservative

$1.00 toll

$1.00 credit
"Energy" conservation

Conservative! (Potential Money)

$1.00 toll

$1.00 credit

Springs (Hooke’s Law)

\[ F = -kx \]

Proportional to displacement from equilibrium

Potential Energy of Spring

\[ \sum \Delta PE = \frac{1}{2} (kx)\Delta x \]

\[ PE = \frac{1}{2} kx^2 \]

Example 5.8

A 0.50-kg block rests on a horizontal, frictionless surface as in the figure; it is pressed against a light spring having a spring constant of \( k = 800 \text{ N/m} \), with an initial compression of 2.0 cm.

b) To what height \( h \) does the block rise when moving up the incline?

3.2 cm

Graphical connection between \( F \) and \( PE \)

\[ \Delta PE = -F\Delta x \]

\[ PE_2 - PE_1 = -\text{Area under curve} \]
Example 5.9a
At point 'A', which are zero?
(a) force
(b) acceleration
(c) force and acceleration
(d) velocity

Example 5.9b
At point 'B', which are zero?
(a) force
(b) acceleration
(c) force and acceleration
(d) velocity
(e) kinetic energy

Example 5.9c
All points for which force is negative (to the left):
(a) C, E and G
(b) B and F
(c) A and I
(d) D and H
(e) D, H and I

Example 5.9d
At point 'D', which are zero?
(a) force
(b) acceleration
(c) force and acceleration
(d) velocity
(e) Velocity and kinetic energy

Example 5.10
A particle of mass \( m = 0.5 \) kg is at a position \( x = 1.0 \) m and has a velocity of \(-10.0 \) m/s. What is the furthest points to the left and right it will reach as it oscillates back and forth? 
0.125 and 3.75 m
Power

- Power is rate of energy transfer
  \[ P = \frac{W}{t} \]

- SI units are Watts (W)
  \[ 1 \text{ W} = 1 \text{ J/s} = 1 \text{ kg} \text{ m}^2 \text{ s}^{-3} \]

- US Customary units are hp (horse power)
  \[ 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W} \]

Example 5.11
An elevator of mass 550 kg and a counterweight of 700 kg lifts 23 drunken 80-kg students to the 7th floor of a dormitory 30 meters off the ground in 12 seconds. What is the power required? (in both W and hp)

\[ 41 \text{ kW} = 55 \text{ hp} \]

Example 5.12
A 1967 Corvette has a weight of 3020 lbs. The 427 cu-in engine was rated at 435 hp at 5400 rpm.

a) If the engine used all 435 hp at 100% efficiency during acceleration, what speed would the car attain after 6 seconds?
b) What is the average acceleration? (in “g”s)

a) 120 mph    b) 0.91g

Example 5.13
Consider the Corvette (w=3020 lbs) having constant acceleration of a=0.91g

a) What is the power when v=10 mph?
b) What is the power output when v=100 mph?

a) 73.1 hp    b) 732 hp

(in real world a is larger at low v)

Example 5.14
A physics professor bicycles through air at a speed of v=36 km/hr. The density of air is 1.29 kg/m³. The professor has cross section of 0.5 m². Assume all of the air the professor sweeps out is accelerated to v.

a) What is the mass of the air swept out by the professor in one second?
b) What is the power required to accelerate this air?

a) 6.45 kg    b) 323 W = 0.432 hp
Example 5.15

If the power required to accelerate the air is 40% of the answer from the last problem due to the professor’s sleek aerodynamic shape,

a) what is the power required to accelerate the air?

b) If the professor has an efficiency of 20%, how many kilocalories will he burn in three hours?

DATA: 1 kcal = 4187 J

a) 52.4 W
b) 676 kcal

Power ~ v^3

Since mass swept out is proportional to v, and KE ~ (1/2)mv^2, Power scales as v^3!

If one goes from 35 km/hr to 50 km/hr, power required would rise by 2.91.

Ergometer Demo

Example 5.16

A dam wishes to produce 50 MW of power. If the height of the dam is 75 m, what flow of water is required? (in m^3/s)

68.9 m^3/s = 1.80x10^4 gallons/s

Example 5.16

How much money does it cost to run a 100-W light bulb for one year if the cost of electricity is 8.0 cents/kWh?

$ 70.08

Some energy facts

- US consumes 24% of World's energy (5% of population)
- Each day, each of us consumes:
  - 3 gallons of oil
  - 20 lbs of coal
  - 221 cubic feet of natural gas
- In 2000 the US consumed 9.9x10^{16} BTUs = 1.05x10^{20} J

1 BTU is energy required to raise 1 lb of H_2O 1 degree F
1 BTU = 1055 J
Einstein...

“Rest” energy
\[E = mc^2\]  \(c\) is velocity of light

For small velocities,\n\[E = mc^2 + \frac{1}{2}mv^2\]

For any \(v\),\n\[E = mc^2 \sqrt{1 - \frac{v^2}{c^2}}\]

Example 5.18

Suppose one had a supply of anti-matter which one could mix with matter to produce energy. What mass of antimatter would be required to satisfy the U.S. energy consumption in 2000? (9.9x10^{16} \text{ BTUs})

574 kg