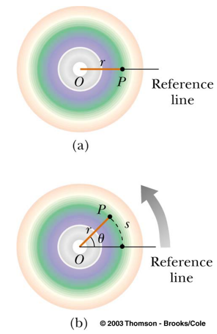
# Chapter 7

Rotational Motion
Universal Law of Gravitation
Kepler's Laws

## Angular Displacement

- Circular motion about AXIS
- Three measures of angles:

1. Degrees
2. Revolutions (1 rev. = 360 deg.)
3. Radians ($2\pi$ rad.s = 360 deg.)

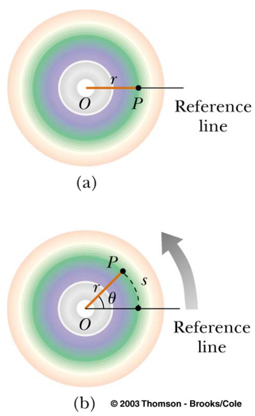




## Angular Displacement, cont.

- Change in distance of a point:

$$s = 2\pi r N \text{ (N counts revolutions)}$$
$$= r\theta \text{ ($\theta$ is in radians)}$$





## Example 7.1

An automobile wheel has a radius of 42 cm. If a car drives 10 km, through what angle has the wheel rotated?

a) In revolutions

b) In radians

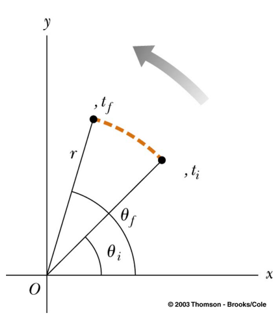
c) In degrees

a) N = 3789

b) $\theta = 2.38\times10^{4}$ radians

c) $\theta = 1.36\times10^{6}$ degrees

## Angular Speed





- Can be given in
  - Revolutions/s
  - Radians/s --> Called $\omega$

$$\omega = \frac{\theta_f - \theta_i}{t} \text{ in radians}$$

- Linear Speed at r

$$v = 2\pi r \cdot \frac{N_{\text{revolutions}}}{t}$$
$$= \frac{\cancel{2\pi} r}{\cancel{2\pi}} \cdot \frac{\theta_f - \theta_i \text{ (in rad.s)}}{t}$$
$$v = \omega r$$

## Example 7.2

A race car engine can turn at a maximum rate of 12,000 rpm. (revolutions per minute).

a) What is the angular velocity in radians per second.

b) If helipcopter blades were attached to the crankshaft while it turns with this angular velocity, what is the maximum radius of a blade such that the speed of the blade tips stays below the speed of sound.
DATA: The speed of sound is 343 m/s

a) 1256 rad/s

b) 27 cm

## Angular Acceleration

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

- Denoted by $\alpha$
- $\omega$ must be in radians per sec.
- Units are rad/$s^2$
- Every point on rigid object has same $\omega$ and $\alpha$

## Rotational/Linear Equivalence:

$$\Delta\theta \leftrightarrow \Delta x$$

$$\omega_0 \leftrightarrow v_0$$

$$\omega_f \leftrightarrow v_f$$

$$\alpha \leftrightarrow a$$

$$t \leftrightarrow t$$

## Linear and Rotational Motion Analogies

| Rotational Motion | Linear Motion |
|---|---|
| $\Delta\theta = \frac{(\omega_0 + \omega_f)}{2} t$ | $\Delta x = \frac{(v_0 + v_f)}{2} t$ |
| $\omega_f = \omega_0 + \alpha t$ | $v_f = v_0 + at$ |
| $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $\Delta x = v_0 t + \frac{1}{2} a t^2$ |
| $\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$ | $\Delta x = v_f t - \frac{1}{2} a t^2$ |
| $\frac{\omega_f^2}{2} = \frac{\omega_0^2}{2} + \alpha\Delta\theta$ | $\frac{v_f^2}{2} = \frac{v_0^2}{2} + a\Delta x$ |

## Example 7.3

A pottery wheel is accelerated uniformly from rest to a rate of 10 rpm in 30 seconds.

a.) What was the angular acceleration? (in rad/$s^2$)

b.) How many revolutions did the wheel undergo during that time?

a) 0.0349 rad/$s^2$

b) 2.50 revolutions

## Linear movement of a rotating point

- Distance
  $x = r\Delta\theta$
- Speed
  $v = r\omega$
- Acceleration
  $a = r\alpha$

Different points have different linear speeds!

Only works for angles in radians!

## Example 7.4

A coin of radius 1.5 cm is initially rolling with a rotational speed of 3.0 radians per second, and comes to a rest after experiencing a slowing down of $\alpha$ = 0.05 rad/$s^2$.

a.) Over what angle (in radians) did the coin rotate?

b.) What linear distance did the coin move?

a) 90 rad
b) 135 cm

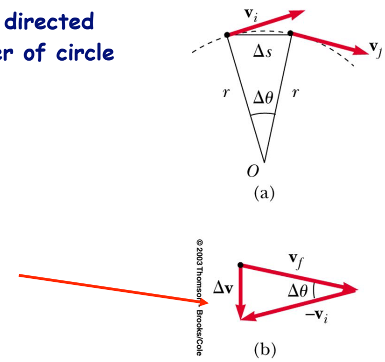
## Centripetal Acceleration

- Moving in circle at constant SPEED does not mean constant VELOCITY
- Centripetal acceleration results from CHANGING DIRECTION of the velocity

## Centripetal Acceleration, cont.

- Acceleration directed toward center of circle

$$\vec{a} = \frac{\Delta \vec{v}}{t}$$




## Derivation: a = $\omega^2$r = v$^2$/r

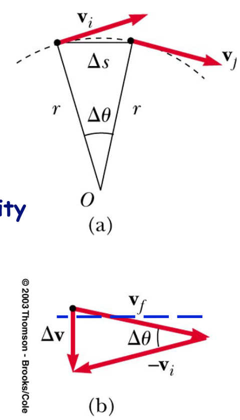
From the geometry of the Figure

$$\Delta v = 2v\sin(\Delta\theta/2)$$
$$= v\Delta\theta \quad \text{for small } \Delta\theta$$

From the definition of angular velocity

$$a = \frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t}$$
$$= v\omega$$

$$a = v\omega = \omega^2 r = \frac{v^2}{r}$$




## Forces Causing Centripetal Acceleration

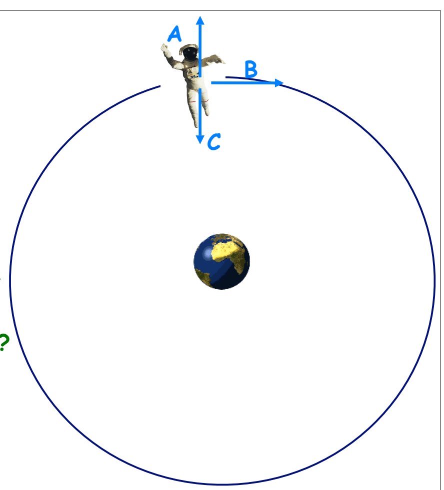
- Newton's Second Law
  $\vec{F} = m\vec{a}$
- Radial acceleration requires radial force
- Examples of forces
  - Spinning ball on a string
  - Gravity
  - Electric forces, e.g. atoms

## Example 7.5a

An astronaut is in cirular orbit around the Earth.

Which vector might describe the astronaut's velocity?

a) Vector A
b) Vector B
c) Vector C

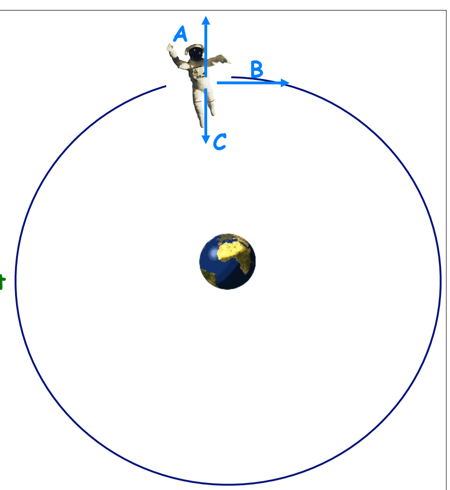



## Example 7.5b

An astronaut is in cirular orbit around the Earth.

Which vector might describe the astronaut's acceleration?

a) Vector A
b) Vector B
c) Vector C

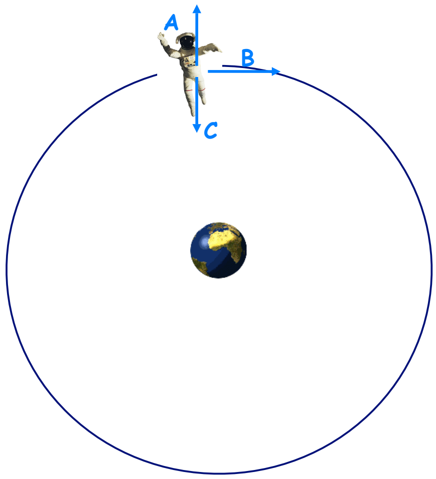


## Example 7.5c

An astronaut is in cirular orbit around the Earth.

Which vector might describe the gravitional force acting on the astronaut?

a) Vector A
b) Vector B
c) Vector C

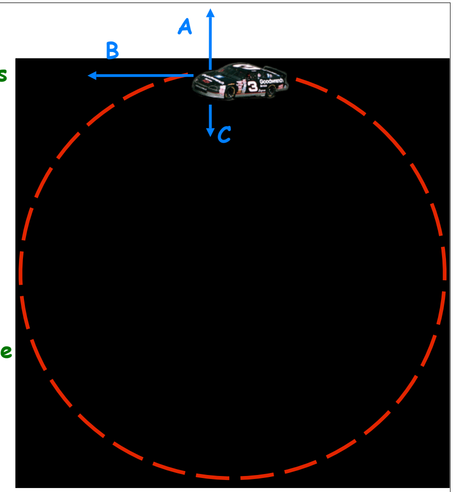



## Example 7.6a

Dale Earnhart drives 150 mph around a circular track at constant speed.

Neglecting air resistance, which vector best describes the frictional force exerted on the tires from contact with the pavement?

a) Vector A
b) Vector B
c) Vector C

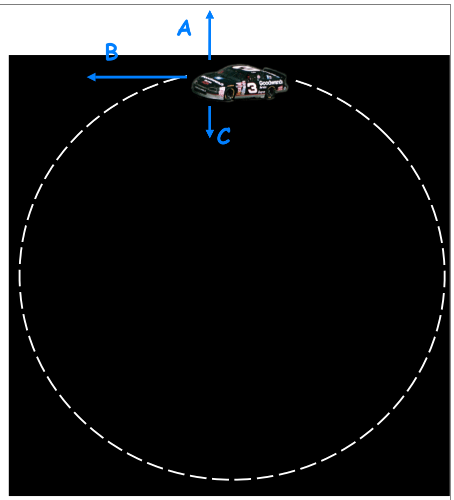



## Example 7.6b

Dale Earnhart drives 150 mph around a circular track at constant speed.

Which vector best describes the frictional force Dale Earnhart experiences from the seat?

a) Vector A
b) Vector B
c) Vector C




## Ball-on-String Demo

## Example 7.7

A space-station is constructed like a barbell with two 1000-kg compartments separated by 50 meters that spin in a circle (r=25 m). The compartments spin once every 10 seconds.

a) What is the acceleration at the extreme end of the compartment? Give answer in terms of "g"s.

b) If the two compartments are held together by a cable, what is the tension in the cable?

a) 9.87 m/s$^2$ = 1.01 "g"s
b) 9870 N

## DEMO: FLYING POKER CHIPS

## Example 7.8

A race car speeds around a circular track.

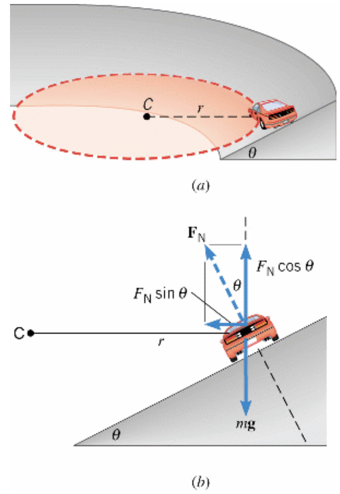
a) If the coefficient of friction with the tires is 1.1, what is the maximum centripetal acceleration (in "g"s) that the race car can experience?

b) What is the minimum circumference of the track that would permit the race car to travel at 300 km/hr?

a) 1.1 "g"s
b) 4.04 km (in real life curves are banked)

## Example 7.9

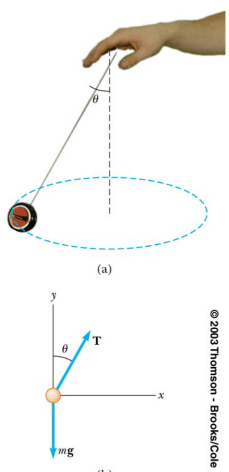
A curve with a radius of curvature of 0.5 km on a highway is banked at an angle of 20°. If the highway were frictionless, at what speed could a car drive without sliding off the road?




42.3 m/s = 94.5 mph

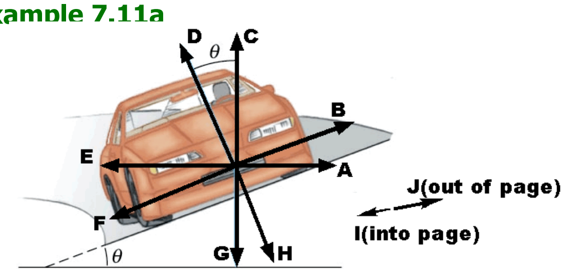
## (Skip) Example 7.10

A yo-yo is spun in a circle as shown. If the length of the string is L = 35 cm and the circular path is repeated 1.5 times per second, at what angle θ (with respect to the vertical) does the string bend?




θ = 71.6 degrees

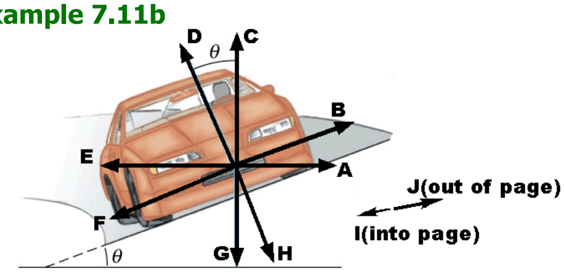
## Example 7.11a




Which vector represents acceleration?

a) A
b) E
c) F
d) B
e) J

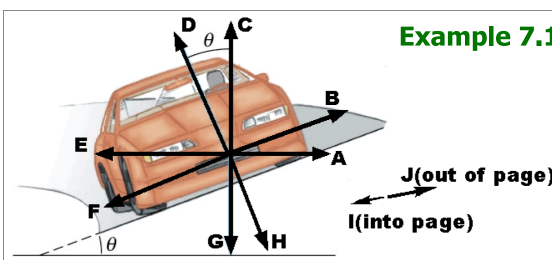
## Example 7.11b




Which vector represents net force acting on car?

a) A
b) E
c) F
d) B
e) J

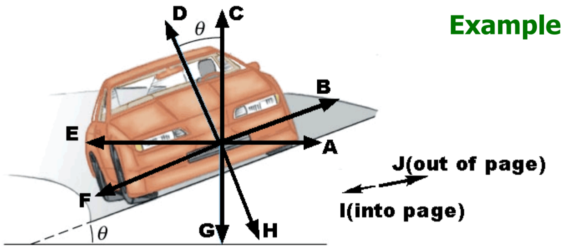
## Example 7.11c




If car moves at "design" speed, which vector represents the force acting on car from contact with road

a) D
b) E
c) G
d) I
e) J

## Example




If car moves slower than "design" speed, which vector represents frictional force acting on car from contact with road (neglect air resistance)

a) B
b) C
c) E
d) F
e) I

## Example 7.12 (skip)

A roller coaster goes upside down performing a circular loop of radius 15 m. What speed does the roller coaster need at the top of the loop so that it does not need to be held onto the track?

12.1 m/s

## Accelerating Reference Frames

$$F = ma$$

$$F - ma_f = m(a - a_f)$$

Fictitious force
Looks like "gravitational" force

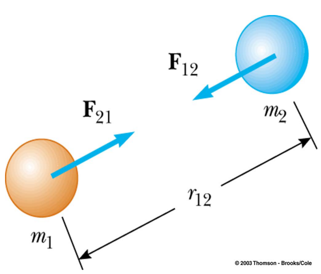
## Example 7.13

Which of these astronauts experiences "weightlessness"?

BOB: who is stationary and located billions of light years from any star or planet.
TED: who is falling freely in a broken elevator.
CAROL: who is orbiting Earth in a low orbit.
ALICE: who is far from any significant stellar object in a rapidly rotating space station

A) BOB & TED
B) TED
C) BOB, TED & CAROL
D) BOB, CAROL & ALICE
E) BOB, TED, CAROL & ALICE

## Newton's Law of Universal Gravitation





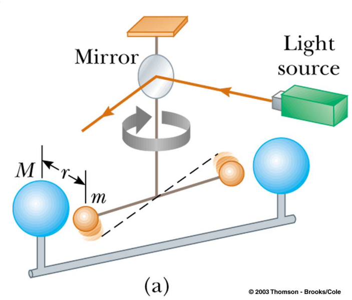
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \left(\frac{\mathrm{m}^3}{kg \cdot s^2}\right)$$

- Always attractive
- Proportional to both masses
- Inversely proportional to separation squared

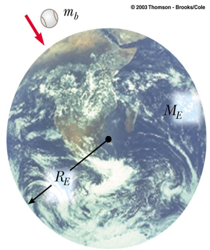
## Gravitation Constant

- Determined experimentally
- Henry Cavendish, 1798
- Light beam / mirror amplify motion



## Example 7.14

Given: In SI units, $G = 6.67 \times 10^{-11}$, $g=9.81$ and the radius of Earth is $6.38 \times 10^{6}$.
Find Earth's mass:




$5.99 \times 10^{24}$ kg

## Example 7.15

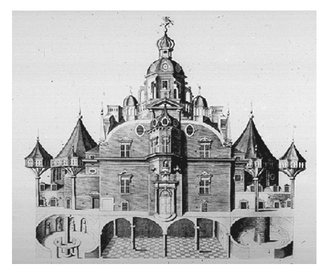
Given: The mass of Jupiter is $1.73 \times 10^{27}$ kg
and Period of Io's orbit is 17 days
Find: Radius of Io's orbit

$r = 1.85 \times 10^{9}$ m

## Tycho Brahe (1546-1601)


- Lost part of nose in a duel
- EXTREMELY ACCURATE astronomical observations, nearly 10X improvement, corrected for atmosphere
- Believed in Retrograde Motion
- Hired Kepler to work as mathematician

## Uraniborg (on an island near Copenhagen)



## Johannes Kepler (1571-1630)

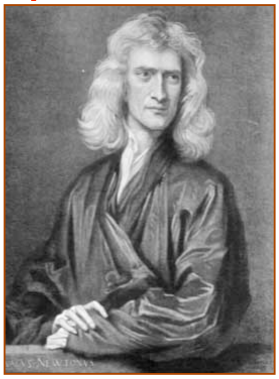
- First to:
  - Explain planetary motion
  - Investigate the formation of pictures with a pin hole camera;
  - Explain the process of vision by refraction within the eye
  - Formulate eyeglass designed for nearsightedness and farsightedness;
  - Explain the use of both eyes for depth perception.

## Johannes Kepler (1571-1630)

- First to:
  - explain the principles of how a telescope works
  - discover and describe total internal reflection.
  - explain that tides are caused by the Moon.
  - suggest that the Sun rotates about its axis
  - derive the birth year of Christ, that is now universally accepted.
  - derive logarithms purely based on mathematics
  - He tried to use stellar parallax caused by the Earth's orbit to measure the distance to the stars; the same principle as depth perception. Today this branch of research is called astrometry.

## Isaac Newton (1642-1727)

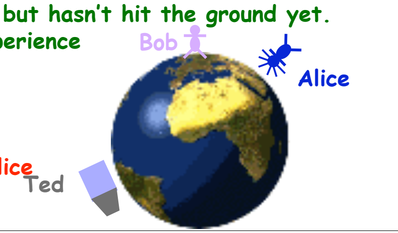
- Invented Calculus
- Formulated the universal law of gravitation
- Showed how Kepler's laws could be derived from an inverse-square-law force
- Invented Wave Mechanics
- Numerous advances to mathematics and geometry



## Example 7.16a

Astronaut Bob stands atop the highest mountain of planet Earth, which has radius R.
Astronaut Ted whizzes around in a circular orbit at the same radius.
Astronaut Carol whizzes around in a circular orbit of radius 3R.
Astronaut Alice is simply falling straight downward and is at a radius R, but hasn't hit the ground yet.
Which astronauts experience weightlessness?

A.) All 4
B.) Ted and Carol
C.) Ted, Carol and Alice


## Example 7.16b

Astronaut Bob stands atop the highest mountain of planet Earth, which has radius R.
Astronaut Ted whizzes around in a circular orbit at the same radius.
Astronaut Carol whizzes around in a circular orbit of radius 3R.
Astronaut Alice is simply falling straight downward and is at a radius R, but hasn't hit the ground yet.
Assume each astronaut weighs w=180 lbs on Earth.

The gravitational force acting on Ted is

A.) w
B.) ZERO

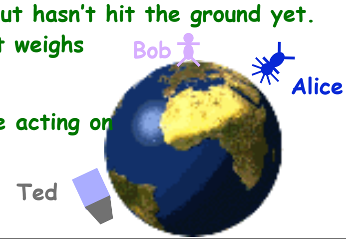


## Example 7.16c

Astronaut Bob stands atop the highest mountain of planet Earth, which has radius R.
Astronaut Ted whizzes around in a circular orbit at the same radius.
Astronaut Carol whizzes around in a circular orbit of radius 3R.
Astronaut Alice is simply falling straight downward and is at a radius R, but hasn't hit the ground yet.
Assume each astronaut weighs w=180 lbs on Earth.

The gravitational force acting on Alice is

A.) w
B.) ZERO


## Example 7.16d

Astronaut Bob stands atop the highest mountain of planet Earth, which has radius R.
Astronaut Ted whizzes around in a circular orbit at the same radius.
Astronaut Carol whizzes around in a circular orbit of radius 3R.
Astronaut Alice is simply falling straight downward and is at a radius R, but hasn't hit the ground yet.
Assume each astronaut weighs w=180 lbs on Earth.

The gravitational force acting on Carol is

A.) w
B.) w/3
C.) w/9
D.) ZERO

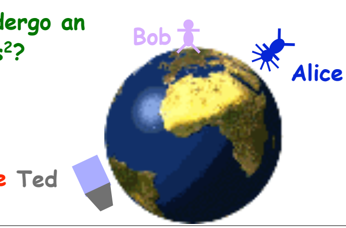


## Example 7.16e

Astronaut Bob stands atop the highest mountain of planet Earth, which has radius R.
Astronaut Ted whizzes around in a circular orbit at the same radius.
Astronaut Carol whizzes around in a circular orbit of radius 3R.
Astronaut Alice is simply falling straight downward and is at a radius R, but hasn't hit the ground yet.

Which astronaut(s) undergo an acceleration $g=9.8\ m/s^2$?

A.) Alice
B.) Bob and Alice
C.) Alice and Ted
D.) Bob, Ted and Alice
E.) All four

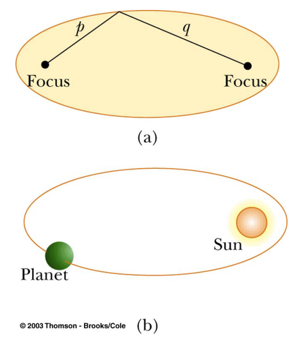



## Kepler's Laws

1) Planets move in elliptical orbits with Sun at one of the focal points.
2) Line drawn from Sun to planet sweeps out equal areas in equal times.
3) The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

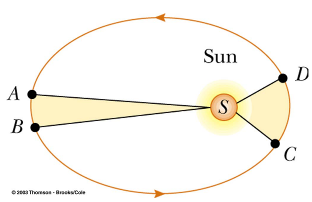
## Kepler's First Law

- Planets move in elliptical orbits with the Sun at one focus.
  - Any object bound to another by an inverse square law will move in an elliptical path
  - Second focus is empty




## Kepler's Second Law

- Line drawn from Sun to planet will sweep out equal areas in equal times
  - Area from A to B and C to D are the same




True for any central force due to angular momentum conservation (next chapter)

## Kepler's Third Law

- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

$$\frac{T^2}{r^3} = K_{\text{sun}}$$

- For orbit around the Sun, $K_S$ = 2.97x10$^{-19}$ s$^2$/m$^3$
- K is independent of the mass of the planet

## Derivation of Kepler's Third Law

$$F = ma = G\frac{Mm}{R^2}$$
$$a = \omega^2 R$$
$$\omega = \frac{2\pi}{T}$$

$$G\frac{Mm}{R^2} = ma$$
$$= m\omega^2 R$$
$$= m\frac{(2\pi)^2}{T^2}R$$
$$\frac{GM}{(2\pi)^2} = \frac{R^3}{T^2}$$

## Example 7.17

Data: Radius of Earth's orbit = 1.0 A.U.
Period of Jupiter's orbit = 11.9 years
Period of Earth's orbit = 1.0 years

Find: Radius of Jupiter's orbit

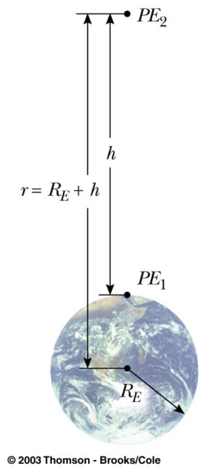
5.2 A.U.

## Gravitational Potential Energy

- PE = mgh valid only near Earth's surface
- For arbitrary altitude

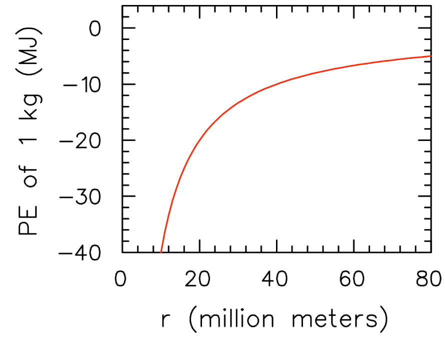
$$PE = -G\frac{Mm}{r}$$

- Zero reference level is at r=∞




## Graphing PE vs. position

$$PE = -G\frac{Mm}{r}$$




## Example 7.18

You wish to hurl a projectile from the surface of the Earth ($R_e$= 6.38x10$^6$ m) to an altitude of 20x10$^6$ m above the surface of the Earth. Ignore rotation of the Earth and air resistance.

a) What initial velocity is required? a) 9,736 m/s

b) What velocity would be required in order for the projectile to reach infinitely high? I.e., what is the escape velocity? b) 11,181 m/s

c) (skip) How does the escape velocity compare to the velocity required for a low earth orbit? c) 7,906 m/s