your name______________________________________

Physics 321 Exam #1 - Friday, Feb. 2, 2018

FYI:
Some integrals:

$$
\begin{aligned}
\int_0^x \frac{dy}{\sqrt{1-y^2}} &= \sin^{-1}(x), \\
\int_0^x \frac{dy}{\sqrt{1+y^2}} &= \sinh^{-1}(x), \\
\int_0^x \frac{dy}{1+y^2} &= \tan^{-1}(x), \\
\int_0^x \frac{dy}{1-y^2} &= \tanh^{-1}(x).
\end{aligned}
$$

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1. A projectile of mass m is launched with velocity v_0 straight-up in a constant gravitational field of acceleration g. The projectile feels a drag force,

$$|F_d| = Av^2.$$

(a) (10 pts) Solve for the speed as a function of time.

(b) (5 pts) How much time is required to reach the apex of the trajectory?

(a)

$$\frac{dv}{dt} = -\frac{Av^2}{m} - g$$

$$t = -\int_{v_0}^{v} \frac{dv'}{g + \frac{A}{m}v'^2} = -\frac{v_c}{g}\int_{v_0/v_c}^{v/v_c} \frac{dx}{1+x^2}, \quad v_c = \sqrt{\frac{mg}{A}}$$

$$= -\sqrt{\frac{m}{gA}}\tan^{-1}(v/v_c) + \sqrt{\frac{m}{gA}}\tan^{-1}(v_0/v_c)$$

$$v = v_c \tan\left[\tan^{-1}(v_0/v_c) - \sqrt{\frac{gA}{m}}\,t\right]$$

(b) $v = 0$ at top

$$t\sqrt{\frac{gA}{m}} = \tan^{-1}(v_0/v_c)$$

$$t = \sqrt{\frac{m}{gA}}\tan^{-1}(v_0/v_c)$$

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2. (15 pts) Nancy has an iceboat which glides on a frictionless lake. The boat is equipped with a really cool slingshot that fires little pebbles. The pebbles are fired with a speed v_p relative to the boat. The boat initially has a mass M_0, which includes the mass of the pebble arsenal, and is sliding along the lake with velocity v_0. Nancy wishes to stop the boat by firing pebbles forward. What mass of pebbles must Nancy fire in order to stop the boat? Give answer in terms of v_0, v_p and M_0.

$$(M - \delta m)(v - \delta v) + \delta m (v + v_p) = Mv$$

$$-M\delta v + \delta m\, v_p = 0$$

$$\int \frac{dm}{m} = \int_{v_0}^{0} \frac{dv}{v_p} = \frac{-v_0}{v_p} = \ln \frac{M_f}{M_0}$$

$$M_f = M_0 e^{-v_0/v_p}$$

$$M_{pebbles} = M_0 - M_f$$

$$= M_0 \left(1 - e^{-v_0/v_p}\right)$$

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3. A particle of mass m moves according to the potential

$$V(x) = -A|x|^\alpha, \quad \text{where } \alpha > 0, \text{ and } A > 0.$$

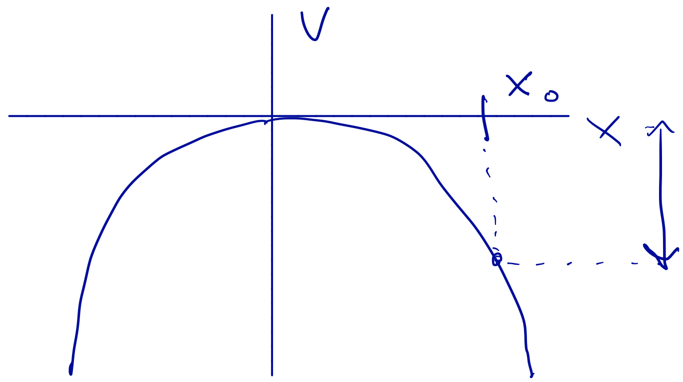
A particle begins at position x_0 with zero total energy (potential plus kinetic) and is aimed toward the origin. It has precisely the amount of initial kinetic energy to stop at $x = 0$.

(a) (5 pts) Sketch the potential, and label x_0. Denote the initial kinetic energy.

(b) (10 pts) How much time is required for the particle to reach the origin?

(c) (5 pts) For what values of α is the answer in (b) finite?

a)



initial KE

$E = \emptyset$

$KE = \frac{1}{2} m v^2$

$v = \frac{2(KE)}{m}$

b) $t = -\int_{x_0}^{0} \frac{dx}{\sqrt{(\cancel{E} + A x^{\alpha})\frac{2}{m}}}$

$= \sqrt{\frac{m}{2A}} \int_0^{x_0} \frac{dx}{x^{\alpha/2}}$

$= \sqrt{\frac{m}{2A}} \left\{ \frac{1}{x_0^{\alpha/2-1}} - \frac{1}{\varepsilon^{\alpha/2-1}} \right\} \frac{-1}{\left(\frac{\alpha}{2}-1\right)}$

$\varepsilon \to 0$

$$= \begin{cases} \sqrt{\frac{m}{2A}} \frac{1}{1-\alpha/2} x_0^{1-\frac{\alpha}{2}}; & \alpha < 2 \\ \infty, & \alpha > 2 \end{cases}$$