your name_____

Physics 321 Exam #2 - Monday, Mar. 25

1. (20 pts) The Rutherford cross section for a particle with kinetic energy E incident on a fixed potential, $V(r) = \alpha/r$, is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2 \sin^4(\theta_s/2)}, \qquad \alpha = \frac{Zze^2}{4\pi\epsilon_0}.$$

Here, Ze and ze are the charges of the target and beam particles respectively. An experiment is designed with N_b beam particles incident on a thin target with ρ_A target particles per area. You place a small detector at a scattering angle of θ_s relative to the beam direction. The face of the detector has an area a and is located a distance $R >> a$, from the target. The detector efficiency is $\kappa =$ the fraction of particles hitting the detector that are recorded.

How many particles are recorded by the detector? Give answer in terms of N_b, α, E, θ_s, R, a, ρ_A and κ. (It is recommended to check your answer is dimensionless as it is a pure number – answers with the wrong dimension will lose at least half credit.)

Prob of hitting detector

$$P_{hit} = \rho_A \cdot \frac{d\sigma}{d\Omega} \cdot \Delta\Omega \;, \quad \Delta\Omega = \frac{a}{R^2}$$

that are recorded

$$N_{rec} = \kappa \cdot N_b \cdot P_{hit}$$

$$= \kappa N_b \rho_A \frac{a}{R^2} \frac{\alpha^2}{16E^2 \sin^4(\theta_s/2)}$$

your name_____

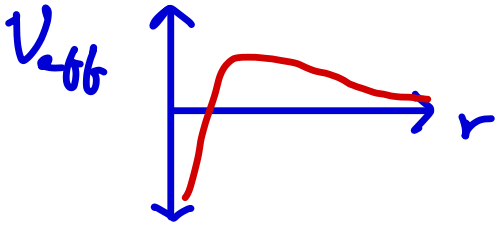
2. A particle of mass m moves according to an attractive potential

$$V(r) = -\frac{\beta}{r^4}.$$

(a) (5 pts) If the particle is in a circular orbit of radius r, what is the angular velocity, $\dot{\theta}$?

$\frac{4\beta}{r^5} = mr\dot{\theta}^2$, $\dot{\theta} = 2\sqrt{\frac{\beta}{mr^6}}$

(b) (5 pts) Sketch the effective radial potential ($V(r)$ plus the centrifugal potential) for a trajectory with angular momentum L.



(c) (5 pts) Is the orbit stable?

NO

(d) (25 pts) A beam of particles with energy E is aimed at the potentential. If the particle reaches the origin, it is annihilated. What is the cross section for annihilation? Give your answer in terms of m, E and β. If your answer does not have dimensions of length-squared, you will lose at least 10 points.

$\sigma_{abs} = \pi b^2 = \pi L^2/2mE$ – need to find L

For grazing traj, $\dot{r} = 0$ at closest point

$E = \frac{L^2}{2mr_{min}^2} - \frac{\beta}{r_{min}^4}$

Find r_{min} to be at max of V_{eff}

$\frac{dV_{eff}}{dr} = 0 \Rightarrow \frac{4\beta}{r_{min}^5} = \frac{L^2}{mr_{min}^3} \Rightarrow r_{min}^2 = \frac{4\beta m}{L^2}$

$E = \frac{L^4}{16m^2\beta}$, $L^2 = 4m\sqrt{\beta E}$

$\sigma_{abs} = 2\pi\sqrt{\beta/E}$

β has dim of energy · length^4 ✓

your name_____

Additional Space for No. 2

your name_____

3. Consider hard spheres of radius a and mass m that elastically scatter with one another once they touch, but do not interact with one another until they touch. Consider a thin target made of such particles and a beam of the same type of particle, with beam energy E.

   (a) (5 pts) What is the distance between the centers of two spheres, $|\vec{r}_1 - \vec{r}_2|$, when the spheres are touching?

   $2a$

   (b) (15 pts) What is the cross section for scattering?

   $\pi (2a)^2 = 4\pi a^2$