your name______________________________________

Physics 321 Practice Exam #1 - Wednesday, Oct 8

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned}
x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \quad \omega' = \sqrt{\omega_0^2 - \beta^2} \qquad \text{(under damped)}\\
x &= A e^{-\beta t} + B t e^{-\beta t}, \qquad \text{(critically damped)}\\
x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).}
\end{aligned}$$

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1. A particle of mass m moves in a potential $V(x) = \beta x$. The particle is initially at $x = 0$ with a kinetic energy T_0 moving to the right.

   (a) (1 pt) What is the maximum distance the particle travels before turning around?

   (b) (3 pts) Solve for the position as a function of time. Only worry about the times before the particle turns around.

   (c) (1 pt) How much time is required for the particle to turn around?

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2. (5 pts) A rocket is in deep space, out of the realm of gravitational fields, when it fires its engines and sends its exhaust backward at a speed of v_e relative to the rocket. The rocket has two stages of equal mass M filled with fuel, plus a payload of mass P. Both stages have a fraction f of their mass remaining after they have expelled all their fuel. Find the final speed of the payload after both stages have expended their fuel.

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3. Consider an underdamped one-dimensional harmonic oscillator of a particle of mass m where the spring constant is k and the drag force is $-bv$.

   (a) (2 pts) Write the general solution in terms of two arbitrary constants.

   (b) (3 pts) Now, add an external force, $F_{\rm ext}(t) = mge^{-\lambda t}\Theta(t)$. Find a particular solution for $t > 0$ that behaves as $\sim e^{-\lambda t}$. ($\Theta(t) = 1$ for $t > 0$, =0 otherwise)

   (c) (3 pts) If at $t = 0$ the particle is at position x_0 with zero velocity, find the exact solution $x(t)$ for $t > 0$.

   (d) (2pts) Consider the limit where the force is very strong, but is applied for a short time. $F_0 \to \infty, \lambda \to \infty$ and $F_0/\lambda = I_0$. For the same initial conditions described above, find $x(t)$ for $t > 0$. Given answer in terms of β, ω_0 and I_0.