Show that $\ddot{x} + 4\dot{x} + 3x = 0$ is overdamped

Solutions are e^{-t} and e^{-3t}

General Solution: $x(t) = A_1 e^{-t} + A_2 e^{-3t}$

Solution shows that it's overdamped because the roots are real and different

Given initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ solve for $x(t)$ and $\dot{x}(t)$

$$A_1 e^{-0} + A_2 e^{-3(0)} = 1$$
$$A_1 + A_2 = 1$$

$$\dot{x}(t) = -A_1 e^{-t} - 3A_2 e^{-3t}$$
$$-A_1 e^{-0} - 3A_2 e^{-3(0)} = 0$$
$$-A_1 - 3A_2 = 0$$

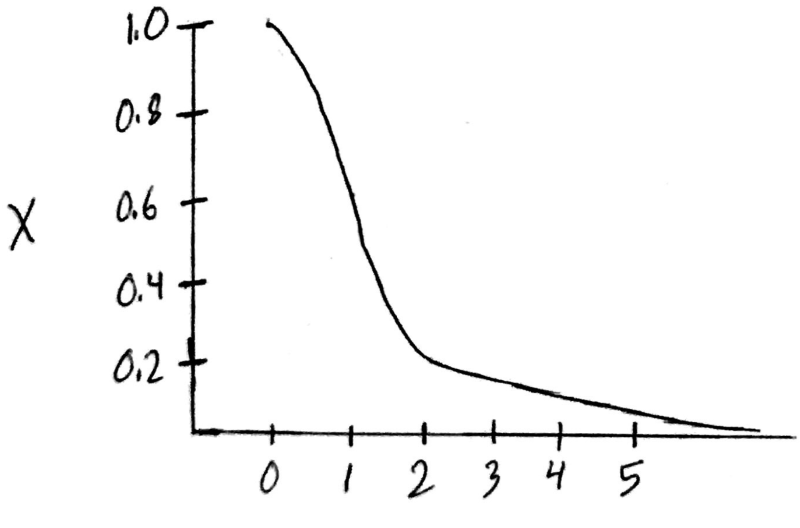
$$A_1 = 1 - A_2 \qquad A_1 = -3A_2$$

$$-3A_2 = 1 - A_2$$
$$-2A_2 = 1$$
$$A_2 = -1/2$$

$$A_1 = 1 - {-1/2}$$
$$A_1 = 3/2$$

$$x(t) = 3/2\, e^{-t} - 1/2\, e^{-3t}$$
$$\dot{x}(t) = -3/2\, e^{-t} + 3/2\, e^{-3t}$$
$$\ddot{x}(t) = 3/2\, e^{-t} - 9/2\, e^{-3t}$$

$$x(t) = (3/2\, e^{-t} - 9/2\, e^{-3t}) + 4(-3/2\, e^{-t} + 3/2\, e^{-3t}) + 3(3/2\, e^{-t} - 1/2\, e^{-3t}) = 0$$
$$x(t) = 3/2\, e^{-t} - 9/2\, e^{-3t} - 6e^{-t} + 6e^{-3t} + 9/2\, e^{-t} - 3/2\, e^{-3t} = 0$$
$$x(t) = e^{-t}(3/2 - 6 + 9/2) + e^{-3t}(-9/2 + 6 - 3/2) = 0$$
$$x(t) = e^{-t}(0) + e^{-3t}(0) = 0$$
$$x(t) = 0 = 0$$



For large t, x approaches 0. This makes sense with our solution