Coriolis Force On a Dropped Object

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Dr. Scott Pratt overhears a student bragging about how good they are at Classical Mechanics and decides to offer an extra credit assignment to test their skills. The student must jump off of a plane heading directly North with a constant velocity and height and land at the Biomedical Physical Sciences Building while accounting for the Coriolis Force. Given the velocity of the plane $v_0 = 75\frac{m}{s}$, the height of the plane $h = 5000m$, acceleration of gravity $g = 9.81\frac{m}{s^2}$, latitude of the BPS being 42.7^0 so $\theta = 47.3^0$, and the angular velocity of Earth's rotation $\omega_0 = \frac{2\pi}{3600*24s}$ find where the student should jump out of the plane relative to their landing spot at the BPS? Ignore air resistance, centrifugal terms, the fact that the student will die without a parachute, and assume that the change in θ from the initial velocity from the plane is negligible.

The equations of motion are:

$$a_x(t) = -2(\omega_y\ v_z - \omega_z\ v_y)$$

$$a_y(t) = -2(\omega_z\ v_x - \omega_x\ v_z)$$

$$a_z(t) = -g - 2(\omega_x\ v_y - \omega_y\ v_x)$$

$$\omega_z = \omega_0\cos(\theta) \quad \omega_y = \omega_0\sin(\theta) \quad \omega_x = 0$$

Terms with v_x can be tossed out since v_x will be very small as well as any terms with ω_x since ω_x is equal to zero. Then v_y can be approximated as v_0 since the change in v_y will be negligible and $v_z = -gt$ can be substituted for v_z. Which gives the following:

$$a_x(t) = -2(-w_0 gt\sin(\theta) - w_0 v_0 cos(\theta))$$

$$a_y(t) = 0$$

$$a_z(t) = -g$$

The equation for a_x can then be integrated with respect to time:

$$v_x(t) = 2\omega_0 \int_0^t (v_0 cos(\theta) + gt' sin(\theta))dt'$$

$$v_x(t) = 2\omega_0\ (t'v_0 cos(\theta) + \frac{1}{2}gt'^2 sin(\theta))\Big|_0^t$$

$$v_x(t) = 2w_0(v_0 t cos(\theta) + \frac{1}{2}gt^2 \sin(\theta))$$

Which can then be integrated with respect to time again to find the deflection in the east-west direction:

$$\delta x(t) = 2\omega_0 \int_0^t (v_0 t' cos(\theta) + \frac{1}{2}gt'^2 sin(\theta))dt'$$

$$\delta x(t) = 2\omega_0 \ (\frac{1}{2}v_0 t'^2 cos(\theta) + \frac{1}{6}gt'^3 sin(\theta))\Big|_0^t$$

$$\delta x(t) = 2\omega_0 \ (\frac{1}{2}v_0 t^2 cos(\theta) + \frac{1}{6}gt^3 \sin(\theta))$$

$$\delta x(t) = \omega_0 \ v_0 t^2 cos(\theta) + \frac{1}{3}\omega_0 \ gt^3 sin(\theta)$$

The equation for displacement in the y direction will be the following since there is no significant change in velocity in the y direction:

$$y(t) = v_0 t$$

Deflections can then be found using $h = \frac{1}{2}gt^2$ to find time.

$$t = \sqrt{\frac{2h}{g}}$$

The deflection in the east-west direction as well as displacement in the north-south direction can be found by plugging in the given values:

$$t = \sqrt{\frac{2 * -4000m}{-9.81\frac{m}{s^2}}} = 28.6s$$

$$\delta x = 7.11m \ (east)$$

$$\delta y = 2141.8m \ (north)$$

The student should jump 2141.8 m to the south and 7.11 m to the west of their intended drop point.