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A small asteroid of mass m is aimed at a heavier planet with mass M and radius R. The asteriod's kinetic energy is E_0 when the asteroid is far away. Find the angle where Rutherford scattering breaks down in terms of R, E, G, M, and m

$$
\begin{aligned}
L &= mv_0 b \\
E &= \frac{1}{2}mv_0^2 \\
E &= \frac{L^2}{2mR^2} - \frac{GMm}{R} \\
L^2 &= 2EmR^2 + 2GMRm^2 \\
L &= mvb \\
b^2 &= \frac{L^2}{2mE} \\
b^2 &= \frac{2EmR^2 + 2GMRm^2}{2mE} \\
&= R^2 + \frac{GMmR}{E}
\end{aligned}
$$

For $F = \frac{\alpha}{r^2}$:

$$
\sin\frac{\theta_s}{2} = \frac{a}{\sqrt{a^2+b^2}} \qquad a \equiv \frac{\alpha}{2E}
$$

$$
\begin{aligned}
\sin\frac{\theta_s}{2} &= \frac{\frac{GMm}{2E}}{\sqrt{\left(\frac{GMm}{2E}\right)^2 + R^2 + \frac{GMmR}{E}}} \\
&= \frac{\frac{GMm}{2E}}{\frac{GMm}{2E} + R} \\
&= \frac{1}{1 + \frac{2RE}{GMm}} \\
\theta_s &= 2\sin^{-1}\frac{1}{1+\frac{2RE}{GMm}}
\end{aligned}
$$