Momentum Conservation

Scott Pratt and a railcart have a combined mass of M_p. The railcart slides without friction on top of a never ending track. Scott Pratt and the railcart are moving at an initial velocity of V_0.

a.) After the first exam, Scott Pratt decides that the highest scoring students will be picked up on the railcart. There are n students per distance. What is Scott Pratt's speed as a function of the distance traveled? Each student has a mass M_s.

b.) What is Scott Pratt's position as a function of time?

Solution:

a.) $M_p V_0 = (M_p + M_s \cdot n \cdot x)V$ ← solve for v

$$v = \frac{M_p V_0}{M_p + M_s \cdot n \cdot x}$$

b.) $$\frac{dx}{dt} = \frac{M_p V_0}{M_p + M_s \cdot n \cdot x} \quad \text{(from part a.)}$$

$$M_p V_0 t = \int_0^x M_p + M_s \cdot n \cdot \tilde{x}\, d\tilde{x}$$

$$= M_p \tilde{x} + \frac{1}{2} M_s \cdot n \cdot \tilde{x}^2 \Big|_0^x$$

$$M_p V_0 t = M_p x + \frac{1}{2} M_s \cdot n x^2 \longrightarrow \frac{1}{2} M_s \cdot n x^2 + M_p x - M_p V_0 t = 0$$

By using the quadratic equation...

$$x = \frac{-M_p + \sqrt{M_p^2 + 2 M_s \cdot n \cdot M_p \cdot V_0 t}}{M_s n}$$