PHY 321 Final Project
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1 Lagrangian with Angular Momentum:

A fishbowl exists as a hollow sphere of radius R with a circular opening starting at $\theta = 45$ degrees up from the center. An ice cube (frictionless) is placed on the inside surface of the bowl halfway up, and shoved with some initial velocity $\vec{v_0} =< v_x, v_y, v_z >$ while it spins around the bowl with azimuthal angle ϕ.

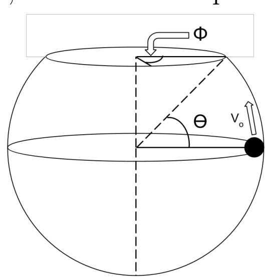
A. Find the Lagrangian of this system, as the ice cube spins and oscillates up and down along the edge

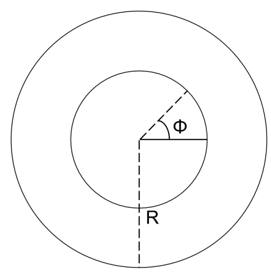
B. Find the equations of motion for the ice cube.

C. Replace $\dot{\phi}$ with an expression of angular velocity in the equation of motion

2 Answers:

A. First, let's draw some pictures, front center and top down:





The System has 2 degrees of freedom (θ and ϕ) and thus, both kinetic and Potential energy will have 2 components. Let's start with kinetic:

Step 1 - Find T: To find each T component, we need the velocity of each component

$$v_\phi = r * \dot{\phi}$$

$$r = R * cos(\theta)$$

$$T_\phi = \frac{m}{2}(v_\phi)^2$$

so we can say:

$$T_\phi = \frac{m}{2}R^2 cos^2(\theta)\dot{\phi}^2$$

Now for the other component:

$$v_\theta = R * \dot{\theta}$$

so we can say:

$$T_\theta = \frac{m}{2} R^2 \dot{\theta}^2$$

Thus, total T is the sum of both components:

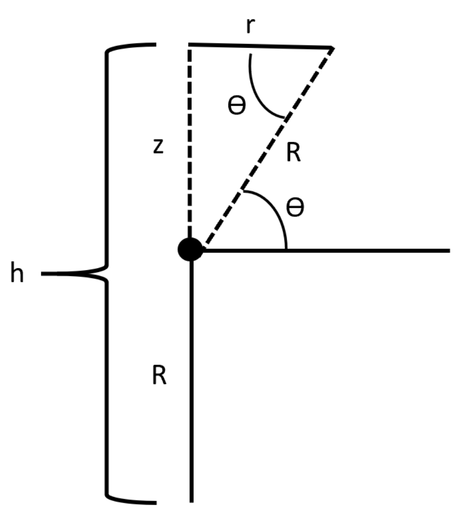
$$T = T_\phi + T_\theta$$

$$T = \frac{m}{2} R^2 cos^2(\theta) \dot{\phi}^2 + \frac{m}{2} R^2 \dot{\theta}^2$$

Step 2 - Find U:

$$U_\phi = 0$$

$$U_\theta = m * g * h$$



$$h = R + z$$

$$z = R * sin\theta$$

$$h = R(1 + sin\theta)$$

$$U = mgR(1 + sin\theta)$$

$$\boldsymbol{L} = \frac{m}{2}R^2cos^2(\theta)\dot{\phi}^2 + \frac{m}{2}R^2\dot{\theta}^2 - mgR(1 + sin\theta)$$

B. We know that:

$$\frac{\delta \boldsymbol{L}}{\delta \dot{\theta}} = mR^2\dot{\theta}$$

$$\frac{d}{dt}(\frac{\delta \boldsymbol{L}}{\delta \dot{\theta}}) = mR^2\ddot{\theta}$$

$$\frac{\delta \boldsymbol{L}}{\delta \theta} = -mR^2\dot{\phi}^2cos\theta sin\theta - mgRcos\theta$$

$$\frac{d}{dt}(\frac{\delta \boldsymbol{L}}{\delta \dot{\theta}}) = \frac{\delta \boldsymbol{L}}{\delta \theta}$$

$$\ddot{\theta} = -\dot{\phi}^2cos\theta sin\theta - \frac{gcos\theta}{R}$$

C. Angular momentum (L, not to be confused with $\boldsymbol{L}$, the Lagrangian) can be given by:

$$L = m * v * r = m * (Rcos\theta\dot{\phi}) * (Rcos\theta)$$

$$L = mR^2cos^2\theta\dot{\phi}$$

using the equation:

$$\frac{d}{dt}(\frac{\delta \boldsymbol{L}}{\delta \dot{\theta}}) = \frac{\delta \boldsymbol{L}}{\delta \theta}$$

we can say that:

$$\frac{d}{dt}(mR^2cos^2\theta\dot{\phi}) = 0$$

Therefore, L is conserved for all values.

$$\dot{\phi} = \frac{L}{mR^2cos^2\theta}$$

$$\ddot{\theta} = -\frac{L^2sin\theta}{m^2R^4cos^3\theta} - \frac{gcos\theta}{R}$$