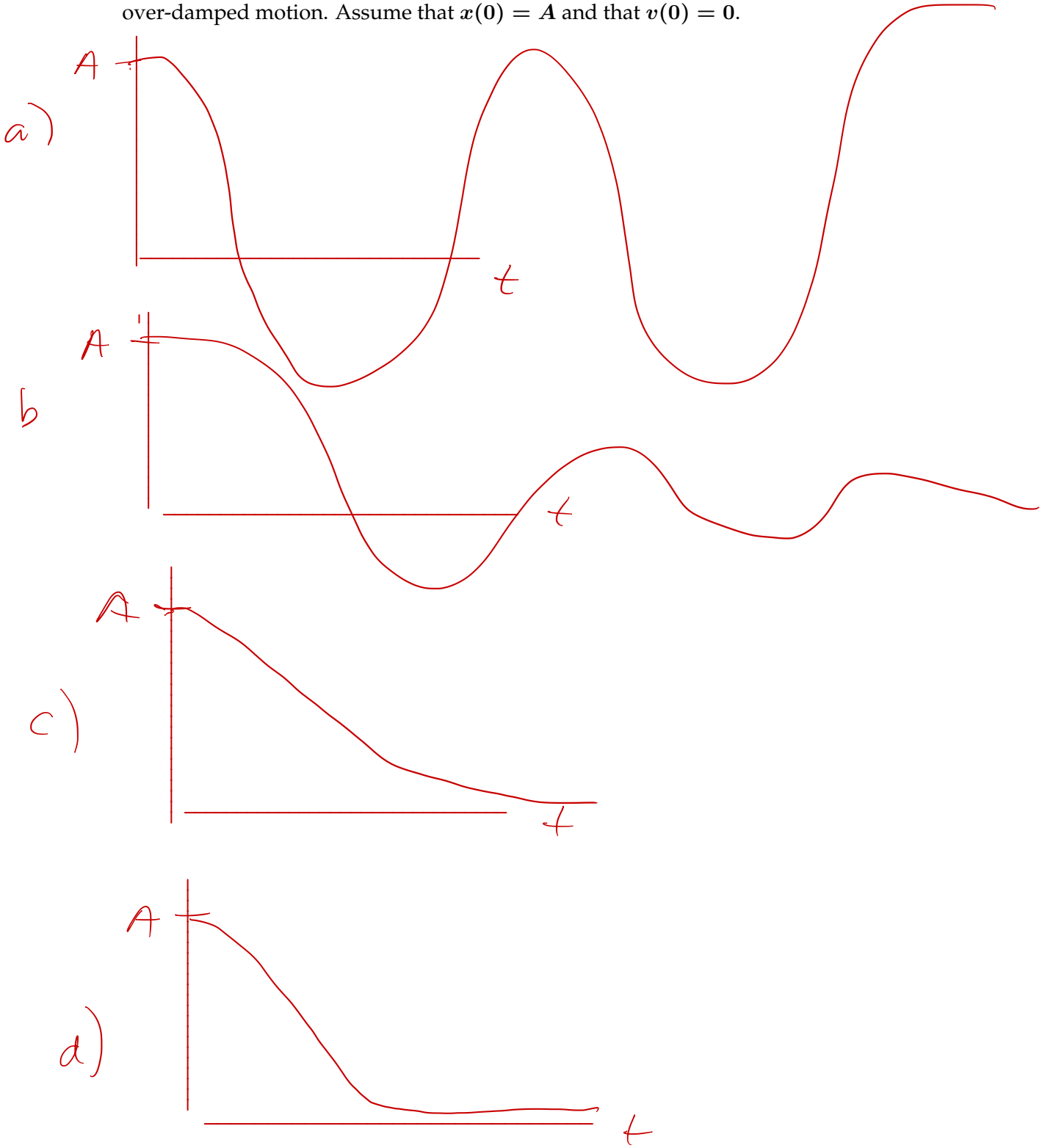
your name(s) KEY

Physics 321 Quiz #1 (Diagnostic, not graded) - Friday, Jan. 12

1. Consider a damped one-dimensional harmonic oscillator where the potential is centered at $\boldsymbol{x = 0}$. Make qualitative graphs of $\boldsymbol{x(t)}$ for (a) undamped (b) under-damped (c) critically-damped and (d) over-damped motion. Assume that $\boldsymbol{x(0) = A}$ and that $\boldsymbol{v(0) = 0}$.



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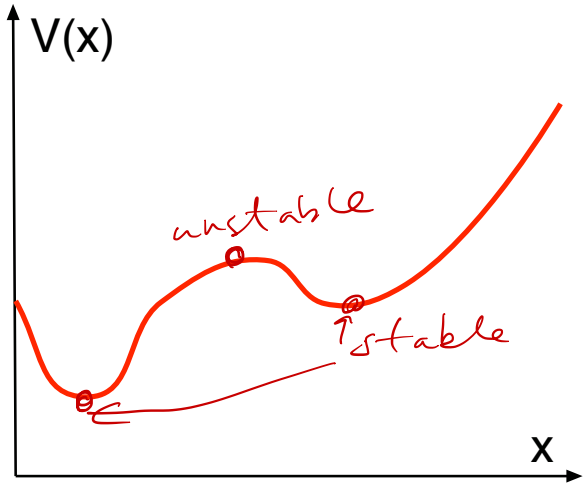
2. Consider the displayed potential as a function of x. (a) Label the points at which there are stable equilibria. (b) Label the points at which there are unstable equilibria.



3. A particle moves according to a cylindrically symmetric attractive potential,

$$V(x, y, z) = \frac{-\kappa}{\rho^3}, \quad \rho \equiv \sqrt{x^2 + y^2}.$$

Which quantities are constants of the motion (don't change with time)? $\vec{L}$ denotes angular momentum, $\vec{p}$ the momentum, and T is the kinetic energy.

(a) p_x
(b) p_y
(c) p_z
(d) T
(e) $E = T + V$
(f) L_x
(g) L_y
(h) L_z
(i) $L^2 = L_x^2 + L_y^2 + L_z^2$

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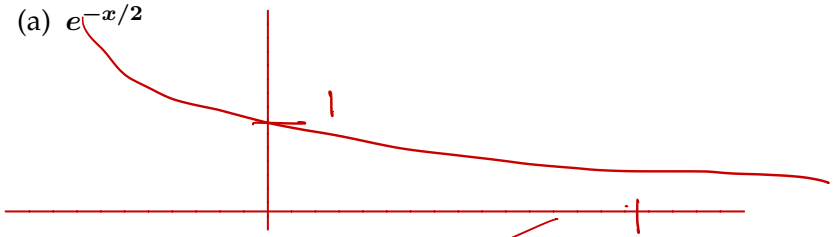
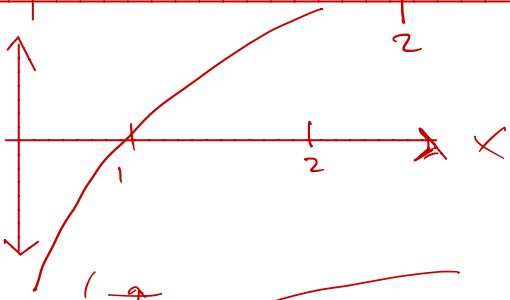
4. Plot the following functions in the region $-2 < x < 2$. DO NOT USE CALCULATOR!!!!

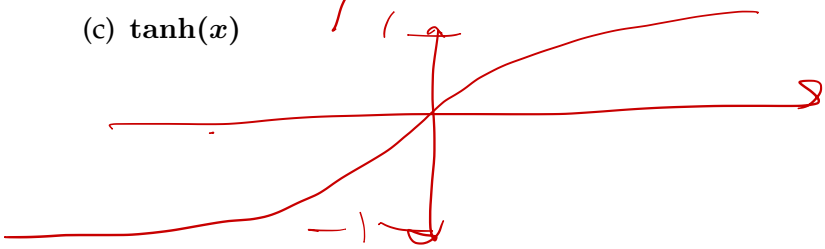
(a) $e^{-x/2}$



(b) $\ln(x)$



(c) $\tanh(x)$



5. A particle of mass m moves in an undamped harmonic oscillator with spring constant k centered at $x = 0$. If the particle is initially at the origin, $x(t = 0) = 0$ and has initial velocity $v(t = 0) = v_0$, write down $x(t)$.

$x = \frac{v_0}{\omega} \sin(\omega t)$, $\omega = \sqrt{k/m}$

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6. Show that:

$$\frac{1}{2}\ln\left[\frac{1+x}{1-x}\right] = \tanh^{-1} x.$$

$$\tanh\underbrace{\left\{\ln\left[\frac{1+x}{1-x}\right]^{1/2}\right\}}_{a} \stackrel{?}{=} x$$

$$\frac{e^{a}-e^{-a}}{e^{a}+e^{a}} \stackrel{?}{=} x$$

$$\frac{\left(\frac{1+x}{1-x}\right)^{1/2}-\left(\frac{1-x}{1+x}\right)^{1/2}}{\left(\frac{1+x}{1-x}\right)^{1/2}+\left(\frac{1-x}{1+x}\right)^{1/2}} \stackrel{?}{=} x$$

$$\frac{\frac{1+x}{\sqrt{1-x^2}}-\frac{1-x}{\sqrt{1-x^2}}}{\frac{1+x}{\sqrt{1-x^2}}+\frac{1-x}{\sqrt{1-x^2}}} \stackrel{?}{=} x$$

$$\frac{2x}{2} \stackrel{?}{=} x \quad \checkmark$$