your name___________________________

Physics 321 Quiz #3 - Wednesday, Sep. 23

1. Ted and his iceboat have a combined mass M_t. Ted's boat slides without friction on the top of a frozen lake. Ted's boat has a winch and he wishes to wind up a long heavy rope of mass M_r and length L that is laid out in a straight line on the ice. Ted's boat starts at rest at one end of the rope, then brings the rope on board the ice boat at a constant rate of R, where R gives the length of the rope that is brought on board per unit time. After a time $t = L/R$ the rope is all aboard the iceboat.

   (a) (5 pts) What is the position of the center of mass of the Ted+rope+boat system relative to Ted

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   The boat doesn't contribute because it is at position $x = 0$ and the rope's center-of-mass is at $L/2$.

$$x_{\rm CM} = \frac{L}{2}\frac{M_r}{M_r + M_t}$$

   (b) (5 pts) Solve for Ted's speed as a function of time

   Use conservation of momentum and the fact that $v_t - v_r = R$,

$$\begin{aligned}
\left(M_t + M_r\frac{Rt}{L}\right) v_t + M_r\left(1 - \frac{Rt}{L}\right) v_r &= 0\\
\left(M_t + M_r\frac{Rt}{L}\right) v_t + M_r\left(1 - \frac{Rt}{L}\right)(v_t - R) &= 0\\
v_t &= \frac{M_r R(1 - Rt/L)}{M_t + M_r}.
\end{aligned}$$

   (c) (5 pts) Solve for Ted's displacement as a function of time. Compare the final position to the answer in (a)

$$\begin{aligned}
x_t(t) &= \int_0^t dt' v_t(t')\\
&= \frac{M_r R t}{M_t + M_r} - \frac{1}{2}\frac{M_r R^2 t^2/L}{M_t + M_r},\\
x_t(t = L/R) &= \frac{L}{2}\frac{M_r}{M_t + M_r}.
\end{aligned}$$

   (d) (5 pts) Let y reference a position on the rope relative to the ice boat. Thus, $0 < y < L - Rt$. Find the tension τ in the rope as a function of y and the time t.

   The rope moves with velocity and acceleration,

$$\begin{aligned}
v_r &= v_t - R = \frac{M_r R(1 - Rt/L)}{M_t + M_r} - R\\
&= -\frac{M_t + M_r Rt/L}{M_t + M_r} R,\\
\frac{dv_r}{dt} &= -\frac{M_r R^2/L}{M_t + M_r}
\end{aligned}$$

The mass beyond point y is the fraction of the rope beyond y multiplied by the rope mass

$$\begin{aligned} m &= M_r(L - Rt - y)/L, \\ \tau &= ma = \frac{M_r^2 R^2 (L - Rt - y)/L}{M_t + M_r}, \quad y < L - Rt. \end{aligned}$$