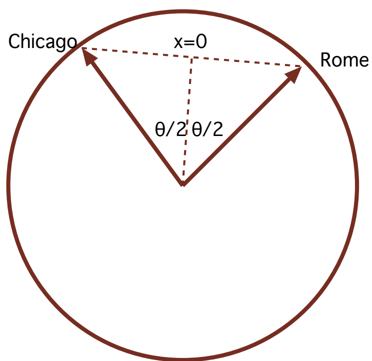
your name______________________________

Physics 321 Quiz #4 - Wednesday, Oct. 14

For this exercise, work in groups of 4. You can use the internet all you want. You may wish to assign different parts of the problem to certain members of your group.
It is year 2525, and if Man is still alive, the great^{14}th grandson of Elon Musk is designing a tunnel that goes directly from Chicago to Rome, Italy (they have basically the same latitude). They wish to understand what would happen if they lose power and need to send supplies to a stuck train somewhere in the tunnel. In this case, only gravity and frictional forces (e.g. air resistance) come into play.



1. If Chicago is at $-x_{\rm max}$ and Rome is at $x_{\rm max}$, find $x_{\rm max}$.
2. Solve for the angular frequency, ω_0, for motion bobbing back and forth around the low point assuming there is no friction.
3. If the supplies experience a linear drag force $-bv$ such that $\beta = (b/2m) = 1.0$ min^{-1}, solve for $x(t)$ for supplies that are dropped in the tunnel from Rome. Be careful to first determine whether this is overdamped or underdamped.
4. What is the position at which the supplies stop and turn around?
5. Plot the speed as a function of x for the supplies as they fall downward.
6. On the same plot, graph the local "terminal" velocity as a function of x.

Solutions:

1)

$$\begin{aligned}
(x_1, y1, z_1) &= R(\cos(41), 0, \sin(41)),\ \ (x_2, y_2, z_2) = R(\cos(41)\cos(100), \cos(41)\sin(41), \sin(41)),\\
\cos(\Delta\phi) &= \frac{1}{R^2}\cos^{-1}(x_1x_2 + y_1y_2 + z_1z_2),\ \ \Delta\phi = 70 \text{ deg.}\\
x_{\rm max} &= R\sin(\Delta\phi/2) = 3677 \text{ m.}
\end{aligned}$$

2)

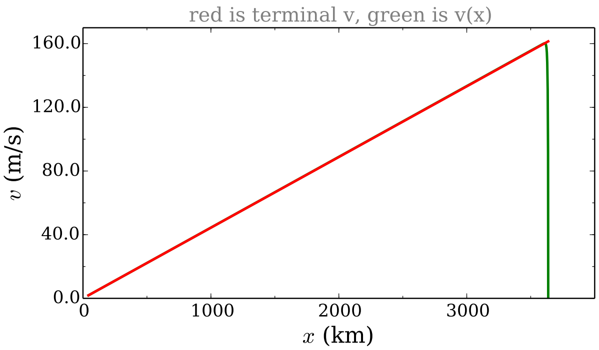
$$F = mg(r/R)\sin\theta = mgx/R,\ \omega_0 = \sqrt{k/m} = \sqrt{g/R} = 0.00124\ \text{s}^{-1}.$$

3)

$$\begin{aligned}
\beta &= 1.0/60\ s^{-1},\\
\beta_1 &= \beta - \sqrt{\beta^2 - \omega_0^2},\ \ \beta_2 = \beta + \sqrt{\beta^2 - \omega_0^2},\\
x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t},\ \ \ x_{\rm max} = A_1 + A_2,\\
v &= -\beta_1 A_1 e^{-\beta_1 t} - \beta_2 A_2 e^{-\beta_2 t},\ \ \ 0 = -\beta_1 A_1 - \beta_2 A_2,\\
A_1 &= \frac{\beta_2 x_{\rm max}}{\beta_2 - \beta_1},\ \ A_2 = \frac{\beta_1 x_{\rm max}}{\beta_1 - \beta_2}
\end{aligned}$$

4) They don't turn around, they just approach stopping at $x = 0$

5&6)



Local terminal velocity is $mg(r)/b = g(r)/(2\beta)$, and nearly matches true speed once particles fall a few hundred meters.