your name_____

Physics 321 Quiz #5 - Wednesday, Oct. 21

Work in groups of 2. Note this problem is HW problem #6 from Chapter 4 of the lecture notes, and is part of next week's assignment. Consider a particle of mass m moving in a potential

$$U = \alpha \ln(r/a).$$

a.) If the particle is moving in a circular orbit of radius R, find the angular frequency $\dot{\theta}$. Solve this by setting $F = m\dot{\theta}^2 r$.

b.) Express the angular momentum L in terms of α, m and R. Also express R in terms of L, α and m.

c.) Sketch the effective radial potential, $V_{\text{eff}}(r)$, for a particle with angular momentum L. (No longer necessarily moving in a circular orbit.)

d.) Find the position of the minimum of V_{eff} in terms of L, α and m, then compare to the result of (b).

e.) What is the effective spring constant for a particle at the minimum of V_{eff}? Express your answer in terms of L, m and α.

f.) What is the angular frequency, ω, for small oscillations of r about the R_{min}? Express your answer in terms of $\dot{\theta}$ from part (a).

Solution:

a)

$$\begin{aligned} F &= -\partial_r U(R) = -\frac{\alpha}{R} = -m\dot{\theta}^2 R, \\ \dot{\theta} &= \sqrt{\frac{\alpha}{mR^2}}. \end{aligned}$$

b)

$$\begin{aligned} L &= mR^2\dot{\theta} = \sqrt{m\alpha R^2}, \\ R &= \sqrt{\frac{L^2}{m\alpha}}. \end{aligned}$$

c)

$$V_{\text{eff}} = \alpha \ln(r/a) + \frac{L^2}{2mr^2}$$

d)

$$\begin{aligned} \frac{d}{dr}V_{\text{eff}} &= 0 = \frac{\alpha}{r} - \frac{L^2}{mr^3}, \\ R_{\text{min}} &= \sqrt{\frac{L^2}{m\alpha}}. \checkmark \end{aligned}$$

e)

$$k_{\text{eff}} = \frac{d^2}{dr^2}V_{\text{eff}}\bigg|_{R_{\text{min}}} = -\frac{\alpha}{R_{\text{min}}^2} + \frac{3L^2}{mR_{\text{min}}^4} = 2\frac{m\alpha}{L^2},$$

f)

$$\omega = \sqrt{k_{\text{eff}}}m = \sqrt{2}\frac{\alpha}{L} = \dot{\theta}\sqrt{2}.$$