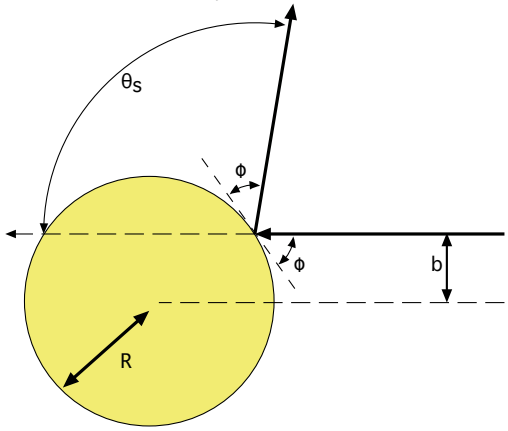
your name_____

Physics 321 Quiz #7 - Wednesday, Nov. 4

Work in groups of 4 or fewer. Feel free to use your lecture notes or book.



A point particle is fired at a spherical target of radius $\boldsymbol{R}$. The particle bounces off the target elastically with scattering angle $\boldsymbol{\theta_s}$. The angle ϕ in the figure is only meant to show that the for a plane tangent to the surface, the angles relative to the surface are equal for the incoming and outgoing trajectories.

1. (1 pt) What is the total cross section $\boldsymbol{\sigma_{\mathrm{tot}}}$?
2. (2 pts) Express the scattering angle $\boldsymbol{\theta_s}$ in terms of the impact parameter $\boldsymbol{b}$.
3. (1 pt) Find the differential cross section $\boldsymbol{d\sigma/d\cos\theta_s}$ as a function of $\boldsymbol{\theta_s}$.
4. (1 pt) Show that the differential cross section integrates to the total cross section.

Solutions:

a) $\boldsymbol{\sigma_{\mathrm{tot}} = \pi R^2}$

b) Let γ be angle from center to collision point of sphere, which bisects incoming and outgoing lines

$$\begin{aligned} b &= R\sin\gamma, \\ \theta_s + 2\gamma &= \pi, \\ b &= R\sin\left(\frac{\pi-\theta_s}{2}\right), \\ &+ R\cos\left(\frac{\theta_s}{2}\right), \\ \theta_s &= 2\cos^{-1}(b/R). \end{aligned}$$

c) (Note should absolute values below)

$$\begin{aligned} d\sigma &= 2\pi b db, \\ &= 2\pi R\cos(\theta_s/2)(R/2)\sin(\theta_s/2)d\theta_s \\ &= \frac{\pi R^2}{2}\sin(\theta_s)d\theta_s \\ &= \frac{\pi R^2}{2}d\cos\theta_s, \\ \frac{d\sigma}{d\cos\theta_s} &= \frac{\pi R^2}{2}. \end{aligned}$$

Note all directions get same scattering flux.
d)

$$\sigma_{\mathrm{tot}} = \int_{-1}^{1} d\cos\theta_s \, \frac{d\sigma}{d\cos\theta_s} \tag{1}$$

$$= \int_{-1}^{1} d\cos\theta_s \frac{\pi R^2}{2} = \pi R^2 \ \checkmark \tag{2}$$

$$\tag{3}$$