your name_____

Physics 321 Quiz #3 - Wednesday, Sep. 17

1. (5 pts) On your homework you derived the relation,

$$\sum_k \epsilon_{ijk}\epsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}.$$

Use this result to prove

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}.$$

$$\begin{aligned}
[\vec{A} \times (\vec{B} \times \vec{C})]_i &= \sum_{jklm} \epsilon_{ijk} A_j \epsilon_{klm} B_l C_m && (1)\\
&= \sum_{jlm} A_j B_l C_m (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) && (2)\\
&= B_i(\vec{A} \cdot C) - C_i(\vec{A} \cdot \vec{B}). && (3)
\end{aligned}$$

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2. (5 pts) A particle of mass m moves in the one-dimensional potential

$$U(x) = \alpha(x - x_0)^2 - \alpha x_0^2,$$

The particle begins at $x = x_0$ moving in the negative direction with kinetic energy $T = \alpha x_0^2$. Solve for the amount of time it takes to reach $x = 0$.

$$\begin{aligned}
E &= U(x_0) + T_{\text{initial}} = \alpha(x_0 - x_0)^2 - \alpha x_0^2 + \alpha x_0^2 = 0 \\
t &= -\int_{x_0}^{0} \frac{dx}{\sqrt{(\alpha x_0^2 - \alpha(x - x_0)^2)(2/m)}} \\
&= -\sqrt{\frac{m}{2\alpha}} \int_0^{-1} \int_0^1 \frac{du}{\sqrt{1 - u^2}}, \qquad u \equiv (x - x_0)/x_0 \\
&= \sqrt{\frac{m}{2\alpha}} \sin^{-1}(1) \\
&= \pi\sqrt{\frac{m}{8\alpha}}.
\end{aligned}$$