your name______________________________________

Physics 321 Quiz #4 - Wednesday, Sep. 24

1. Ted and his iceboat have a combined mass M_t. Ted's boat slides without friction on the top of a frozen lake. Ted's boat has a winch and he wishes to wind up a long heavy rope of mass M_r and length L that is laid out in a straight line on the ice. Ted's boat starts at rest at one end of the rope, then brings the rope on board the ice boat at a constant rate of R, where R gives the length of the rope that is brought on board per unity time. After a time L/R the rope is all aboard the iceboat.

   Let y reference a position on the rope relative to the ice boat. Thus, $0 < y < L - Rt$. Find the tension τ in the rope as a function of y and the time t.

Solution:

From the HW problem,

$$\begin{aligned} v_t - v_r &= R, \\ m_t v_t + m_r v_r &= 0, \\ m_r &= m_{r0} - \lambda Rt, \quad \lambda \equiv m_{r0}/L \\ m_t &= m_{t0} - \lambda Rt. \end{aligned}$$

Solving for the velocity of the rope,

$$v_r = -R\frac{m_{t0} + \lambda Rt}{m_{t0} + m_{r0}}.$$

All elements of the rope outside the boat have the same acceleration,

$$a_r = -\frac{\lambda R^2}{m_{t0} + m_{r0}}.$$

At a position y, the part of the rope outside of "y" has mass

$$m_y = \lambda(L - Rt - y),$$

so the tension is

$$T = m_y|a_r| = \frac{\lambda^2 R^2(L - Rt - y)}{m_{t0} + m_{r0}}.$$

At the end of the rope, $y = L - Rt$, the tension is zero.