your name_____

*Physics 321 Quiz #5 - Wednesday, Oct. 1*

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned} x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \quad \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)} \\ x &= Ae^{-\beta t} + Bte^{-\beta t}, \quad \text{(critically damped)} \\ x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).} \end{aligned}$$

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1. Consider an over-damped harmonic oscillator with a mass of $m = 2$ kg, a damping factor $b = 20$ Ns/m, and a spring constant $k = 32$ N/m. If the initial position is $x = 0.125$ m, and if the initial velocity is $-2.0$ m/s, find and sketch the motion as a function of time. Solve for the time at which the mass crosses the origin.

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**Solution**:
The general solution is

$$x = A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_1 = \beta + \sqrt{\beta^2 - \omega_0^2}, \beta_2 = \beta - \sqrt{\beta^2 - \omega_0^2}.$$

Here $\beta_1 = 8$ and $\beta_2 = 2$. The I.C. give

$$\begin{aligned} 0.125 &= A_1 + A_2, \\ -(\beta_1 A_1 + \beta_2 A_2) &= -2 \\ A_1 &= 7/24, \\ A_2 &= -1/6. \end{aligned}$$

The solution is

$$x = (7/24)e^{-8t} - (1/6)e^{-2t}.$$

This starts above the axis crosses the axis once, then bottoms out and approaches the axis from below. The point it crosses the axis is given by

$$(7/24)e^{-6t} = 1/6, \qquad t = \frac{1}{6}\ln(7/4).$$