your name_____

Physics 321 Quiz #6 - Monday, Oct. 20

You can work in groups of 3 for this quiz. You should turn in one quiz for your group, with all three names. This is closed book.

1. Consider a particle of mass m moving in a radially symmetric potential

$$U = \beta r.$$

   (a) (1 pt) If the particle is in a circular orbit of radius r_0, what is the angular velocity $\dot{\theta}$?

   (b) (1 pt) What is the angular momentum L?

   (c) (2 pts) What is the effective radial potential for such a particle with fixed angular momentum L? Give your answer in terms of L, m, β and r. Sketch the potential.

   (d) (5 pts) Find the angular frequency ω for small oscillations of the radius.

   (e) (1 pt) What is $\omega/\dot{\theta}$?

Solution:

a)

$$|F| = |\partial_r U| = \beta = m r_0 \dot{\theta}^2, \quad \dot{\theta} = \sqrt{\frac{\beta}{m r_0}}.$$

b)

$$L = m r_0^2 \dot{\theta} = \sqrt{\beta m r_0^3}.$$

c)

$$U_{\text{eff}} = \beta r + \frac{L^2}{2mr^2}.$$

d)

$$\begin{aligned} k_{\text{eff}} &= \partial_r^2 U_{\text{eff}}\Big|_{r_0} = \frac{3L^2}{m r_0^4}, \\ \omega = \sqrt{k_{\text{eff}}/m} &= \sqrt{\frac{3L^2}{m^2 r_0^4}} = \sqrt{3}\frac{L}{m r_0^2}. \end{aligned}$$

e)

$$\omega = \dot{\theta}\sqrt{3}.$$

2. (5 pts extra credit) For a potential $U = \beta r^n$, find $\omega/\dot{\theta}$ in terms of n. The answer should only depend on n, not L, β or m. No partial credit!!!

Solution:

$$
\begin{aligned}
k_{\text{eff}} &= \partial_r^2 U_{\text{eff}} = \frac{3L^2}{mr_0^4} + n(n-1)\beta r_0^{n-2}, \\
\frac{L^2}{mr_0^3} &= n\beta r_0^{n-1}, \\
k_{\text{eff}} &= (3+n-1)\frac{L^2}{mr_0^4}, \\
\frac{k_{\text{eff}}}{m} &= (3+n-1)\dot{\theta}^2, \\
\frac{\omega}{\dot{\theta}} &= \sqrt{2+n}.
\end{aligned}
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