your name_____

Physics 321 Quiz #7a - Wednesday, Oct. 29

Work alone for this quiz.
Consider a particle of mass m moving in a radially symmetric attractive potential

$$U = \frac{-\beta}{r^2}.$$

Assume that if, and only if, the particle reaches the origin it will be absorbed.

1. (2 pts) What is the effective radial potential, V_{eff}, for radial motion of a particle with angular momentum L? Sketch $V_{\text{eff}}(r)$ for various values of the angular momentum L.
2. (2 pts) Can the particle have stable circular orbits? Justify your answer.
3. (3 pts) If an incoming particle has energy E and impact parameter b, what is its distance of closest approach?
4. (3 pts) Derive the cross section σ_{abs} for a particle of initial kinetic energy E to be absorbed.

Solutions:
a) $V_{\text{eff}}(r) = L^2/2mr^2 - \beta/r^2$. For $\beta > L^2/2m$ the potential is always negative and for larger β the potential is always > 0.
b) No, there are no minima to the potential.
c)

$$\begin{aligned} E &= V_{\text{eff}}(r_{\text{min}}) = \frac{L^2}{2mr_{\text{min}}^2} - \frac{\beta}{r_{\text{min}}^2} \\ &= E\frac{b^2}{r_{\text{min}}^2} - \frac{\beta}{r_{\text{min}}^2}, \\ r_{\text{min}}^2 &= b^2 - \frac{\beta}{E}. \end{aligned}$$

You can do this alternatively by considering the fact that

$$\begin{aligned} L^2 &= 2mEb^2 = 2m(E + \beta/r_{\text{min}}^2)r_{\text{min}}^2, \\ 2mEb^2 &= 2mEr_{\text{min}}^2 + 2m\beta, \\ r_{\text{min}}^2 &= b^2 - \frac{\beta}{E} \end{aligned}$$

d) The particle will hit the origin if

$$\begin{aligned} L^2 &< 2m\beta, \\ b^2 mE &< 2m\beta \\ b^2 &< \frac{\beta}{E}, \\ \sigma_{\text{abs}} &= \pi\frac{\beta}{E}. \end{aligned}$$