your name(s)______________________________

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Physics 321 Quiz #3 - Friday, Sep. 21

Work in groups of 3. CLOSED BOOK, CLOSED NOTES, OPEN MINDS

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned} x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \;\; \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)} \\ x &= Ae^{-\beta t} + Bte^{-\beta t}, \quad \text{(critically damped)} \\ x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).} \end{aligned}$$

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1. A particle of mass m feels a drag force, $-bv$ and a restoring force $-kx$. Use the definitions, $\beta \equiv b/2m$ and $\omega_0^2 \equiv k/m$. Consider the case where the drag is large, $\beta >> \omega_0$. In the limit $\beta/\omega_0 \to \infty$, solve for the time required for a particle to return half way to the origin, assuming it had zero initial velocity.

$$x = A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}$$

$$\beta_2 \simeq 2\beta, \quad \beta_1 \simeq \beta - \beta\left[1 - \frac{1}{2}\frac{\omega_0^2}{\beta^2}\right] \simeq \frac{\omega_0^2}{2\beta}$$

$$x_0 = A_1 + A_2$$

$$v_0 = -\beta_1 A_1 - \beta_2 A_2 = 0$$

because $\beta_2 \to \infty$, $A_2 = 0$

$$x \simeq x_0 e^{-\omega_0^2 t/2\beta}$$

$$\frac{1}{2} = e^{-\omega_0^2 t_{1/2}/2\beta}$$

$$-\frac{(2\beta \ln 1/2)}{\omega_0^2} = t_{1/2} = \frac{\beta}{\omega_0^2} 2\ln 2$$