your name(s)____________________

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Physics 321 Quiz #4 - Friday, Sep. 28

Work in groups of 3, open notes/book/internet/mouth

FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned}
x &= A_1 e^{-\beta t}\cos\omega' t + A_2 e^{-\beta t}\sin\omega' t \;\; \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)} \\
x &= Ae^{-\beta t} + Bte^{-\beta t}, \quad \text{(critically damped)} \\
x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).}
\end{aligned}$$

A particle of mass m and charge q is in a harmonic oscillator characterized by a restoring force $-kx$ and a small damping force $-bv$. An electric field is applied with the following functional form,

$$\begin{aligned}
E(t) &= \begin{cases} 0, & -\tau/2 < t < 0, \\ E_0, & 0 < t < \tau/2 \end{cases} \\
E(t+\tau) &= E(t)
\end{aligned}$$

A
- (10 pts) Express the electric field as a Fourier series.

B
- (10 pts) Find $x(t)$ assuming the field has been applied for a very long time, much longer than $1/\beta$, and that $\beta << \omega_0$, where $\beta = b/(2m)$ and $\omega_0 = \sqrt{k/m}$. Express your answer as a Fourier series.

C
- (5 pts) For what values of τ will the amplitude become singular?

A) $E = \frac{f_0}{2} + \sum_{n=1}^{\infty} f_n \cos n\omega t + \sum_{n=1}^{\infty} g_n \sin n\omega t$

$f_0 = E_0$, $f_{n>0} = 0$,

$g_n = \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} (E(t) - E_0/2)\, dt \sin n\omega t$

$= \frac{2E_0}{\tau}\int_0^{\tau/2} \sin n\omega t \, dt$

$= \frac{-2E_0}{n\omega\tau}(\cos n\pi - 1) =$

$= \frac{2E_0}{\pi n}$, $n = 1, 3, 5, 7$ $\quad = 0$ otherwise

13) $x_p = A e^{i\omega t}$ for one freq.

$$(-\omega^2 + \omega_0^2) x_{p,n} = \frac{F_0}{m} e^{i\omega t}$$

$$x_p = \frac{F_0/m}{(\omega_0^2 - \omega^2)} \sin \omega t$$

For all frequencies

$$x_p(t) = \frac{qE_0/2}{k} + \sum_{n=1,3,5\cdots} \frac{2qE_0}{\pi n m} \frac{\sin(\omega_n t)}{(\omega_0^2 - \omega_n^2)}$$

$$\omega_n = 2n\pi/\tau$$

c) $= \infty$ when $\frac{2n\pi}{\tau} = \omega_0$

or when $\tau = \frac{2n\pi}{\omega_0}$