your name(s)________________________________

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Physics 321 Quiz #3 - Friday, Feb. 9

Work in groups of 3. CLOSED-BOOK, CLOSED-NOTES
Graded on 20 point scale (15 extra credit points)
FYI:

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta,\\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta,\\ \frac{1}{\cos^2\theta} &= 1+\tan^2\theta.\end{aligned}$$

A particle of mass m moves (in one dimension) through a medium that provides a drag force, $F_d = -bv$. Additionally, the particle experiences an external oscillating force,

$$f = F_0 \sin\omega t.$$

1. (5 pts) For the case where $F_0 = 0$, solve for $v(t)$ with an arbitrary constant, i.e. the homogenous solution $v_h(t)$.
2. (5 pts) Again, with $F_0 = 0$, solve for $x(t)$ given $x(t=0) = 0$ and $v(t=0) = v_0$.
3. (10 pts) Now, consider $F_0 \neq 0$. Solve for the particular solution solution $v_p(t)$.
4. (5 pts) Again assuming $F_0 \neq 0$, given $v(t=0) = v_0$, solve for $v(t)$.
5. (5 pts) Again with $F_0 \neq 0$, given $x(t=0) = 0$ and $v(t=0) = v_0$, solve for $x(t)$.
6. (5 pts) From (5) express $x(t)$ for large times in the form

$$x(t\to\infty) = \bar{X} + A\sin(\omega t+\gamma),$$

i.e. express $\bar{X}$, A and γ in terms of v_0, m, b, and F_0.

1.) $m\dot{v} + bv = 0$

$$v = B e^{-b/m\, t} \quad , B = \text{arb. const.}$$

2.) $v = v_0 e^{-b/m\, t}$

$$x = \int_0^t v\, dt' = \frac{m}{b} v_0 \left(1 - e^{-bt/m}\right)$$

3.) $v_p = V \sin(\omega t + \varphi)$

Find V & φ

$$m\omega V \cos(\omega t + \varphi) + bV \sin(\omega t + \varphi) = F_0 \sin \omega t$$

$$\cos \omega t \left\{ m\omega V \cos\varphi + bV \sin\varphi \right\} = 0$$

$$\sin \omega t \left\{ -m\omega V \sin\varphi + bV \cos\varphi \right\} = F_0 \sin \omega t$$

$$\tan\varphi = -\frac{m\omega}{b}$$

$$\cos\varphi = \frac{1}{\sqrt{1 + \frac{m^2\omega^2}{b^2}}} = \frac{b}{\sqrt{b^2 + m^2\omega^2}}$$

$$\sin\varphi = -\frac{m\omega}{\sqrt{b^2 + m^2\omega^2}}$$

$$V = \frac{F_0}{b\cos\varphi - m\omega \sin\varphi} = \frac{F_0}{\sqrt{b^2 + m^2\omega^2}}$$

4) $v = B e^{-\frac{b}{m}t} + V \sin(\omega t + \varphi)$

$$v_0 = B + V \sin\varphi$$

$$B = v_0 - V \sin\varphi$$

$$v = (v_0 - V\sin\varphi)\, e^{-\frac{b}{m}t} + V \sin(\omega t + \varphi)$$

5) $x = \int_0^t v \, dt'$

$$= \frac{m}{b}(v_0 - V\sin\varphi)\left(1 - e^{-\frac{b}{m}t}\right)$$

$$- \frac{V}{\omega}\left[\cos(\omega t + \varphi) - \cos\varphi\right]$$

6) $x(t \to \infty)$

$$= \frac{m}{b}(v_0 - V\sin\varphi) + \frac{V}{\omega}\cos\varphi$$

$$- \frac{V}{\omega}\cos(\omega t + \varphi)$$

$$\bar{X} = \frac{m}{b}\left(v_0 + V\frac{m\omega}{\sqrt{m^2\omega^2 + b^2}}\right) + \frac{(F_0/\omega)\, b}{(m^2\omega^2 + b^2)}$$

$$A = -\frac{(F_0/\omega)}{\sqrt{m^2\omega^2 + b^2}}$$

$$\varphi = -\tan^{-1}\left(\frac{m\omega}{b}\right)$$

$$x = \bar{X} + A\cos(\omega t + \varphi)$$

$$= \bar{X} + A\sin\left(\omega t + \frac{\pi}{2} - \varphi\right)$$

$$\gamma = \frac{\pi}{2} + \tan^{-1}\frac{m\omega}{b}$$

$$x = \bar{X} + A\sin(\omega t + \gamma)$$