your name(s)______________________________________

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Physics 321 Quiz #5 - Friday, Feb. 23

Consider a spherical asteroid made of rock with density $\rho_a = 2$ g/cm^3 and radius a. It is in a circular orbit of radius r about Saturn. A pebble of mass δm lies on the surface of the asteroid, either at a point facing, or opposite the direction of Saturn.

1. In terms of G, M_s, r, a and δm, express the tidal force on the pebble. The tidal force is the difference between the gravitational force from Saturn acting on the pebble vs the force it would experience if it were located at the center of the asteroid. Keep only the first order term when expanding in terms of a. Note that the tidal force is trying to lift the pebble from the surface.

2. In terms of G, a, δm and ρ_a, what is the gravitational force acting on the pebble due to the gravitational interaction with the asteroid.

3. Equate the two forces to find the radius r at which the tidal force rips apart the asteroid. Saturn's mass is $M_s = 5.68 \times 10^{26}$ kg. Note that $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. Compare your answer to the radius of Saturn, $R_s = 5.82 \times 10^4$ km.

4. What would happen if the asteroid were made of ice instead of rock?

5. Express your answer for δ r in terms of R_s, ρ_a and the density of Saturn ρ_s. Note the density of Saturn is 0.9 gm/cm^3. It would float in your bathtub, if your bathtub were large enough. The average density of Jupiter is 1.3 g/cm^3 and the average density of Earth is 5.5 g/cm^3.

6. If a small moon and planet slowly spiral toward each other, and if the moon and planet have the same density, will the moon be torn apart by tidal forces before reaching the surface of the planet?

1. $F_{tidal} = \frac{dF}{dz} z = \frac{-2GM_s\,\delta m}{r^3}a$

2. $F_{asteroid} = \frac{GM_a\,\delta m}{a^2} = \frac{4\pi}{3}\frac{G\rho_a\,\delta m}{a^2}a^3$

3. $\frac{4\pi}{3}\rho_a = \frac{2}{r^3}$, $r = \left(\frac{3M_s}{2\pi\rho_a}\right)^{1/3} = 5.14\cdot 10^4$ km

4. $\rho_a \sim 1.0$, so r would be larger, and outside atmosphere. Note: rings are mainly ice

5. $M_s = \frac{4\pi}{3}\rho_s R_s^3$, $r = \left(2\frac{\rho_s}{\rho_a}\right)^{1/3} R_s$

6. yes, because $2\rho_{planet} > \rho_{asteroid}$ and $r > R_{planet}$