your name(s)____

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Physics 321 Quiz #6 - Friday, March 2

You can work in groups of up to 3 for this quiz. You should turn in one quiz for your group, with all three names. This is open book.

1. Consider a particle of mass m moving in a radially symmetric potential

$$U(r) = V_0 r^\alpha.$$

   (a) (5 pts) If $V_0 > 0$, for what values of α can you have stable orbits? $\alpha > 0$

   (b) (5 pts) If $V_0 < 0$, for what values of α can you have stable orbits? no stable orbits

2. Consider a particle of mass m moving in a radially symmetric potential

$$U(r) = \frac{V_0}{r^\alpha}.$$

   (a) (5 pts) If $V_0 > 0$, for what values of α can you have stable orbits? no stable orbits

   (b) (5 pts) If $V_0 < 0$, for what values of α can you have stable orbits? $\alpha < 2$

3. Consider a particle of mass m moving in an attractive radially symmetric potential

$$U(r) = -\frac{V_0}{r^2}.$$

   If the particle reaches $r = 0$, it is annihilated.

   (a) (5 pt) Are there any stable orbits? No

   (b) (5 pts) For a particle with kinetic energy T, what is the cross section for a death spiral, σ_{death}?

   (c) (30 extra credit quiz points, all or none) For a trajectory with impact parameter b, solve for the trajectory $r(\theta)$, using a coordinate system where the initial angle is $\theta = 0$. Express your answer in terms of b and $a^2 \equiv |(V_0/E) - (L^2/2m)|$. Give answers for three cases:

   - (a) for trajectories with $b < b_{\text{crit}}$, where annihilation occurs
   - (b) for trajectories with $b > b_{\text{crit}}$, where annihilation doesn't happen
   - (c) for the trajectory with $b = b_{\text{crit}}$.

   This work should be done independently (no groups).

b) $\frac{L^2}{2m} = V_0 \qquad \frac{mv^2 b^2}{2} = V_0$

$\pi b^2 = \sigma_{crit} = \left(\pi V_0 / E\right)$

e) $$\frac{dr}{d\theta} = \frac{\dot r}{\dot\theta} = \frac{-\sqrt{\frac{2E}{m}\left(1+\frac{a^2}{r^2}\right)}}{L/mr^2} = \frac{-v_0\sqrt{1+a^2/r^2}}{L/mr^2}$$

$$a^2 = \left|\frac{V_0}{E} - \frac{L^2}{2mE}\right| = \left|\frac{V_0}{E} - b^2\right|$$

$$v_0 = \sqrt{2E/m}\ , \quad L = mv_0 b$$

$$\frac{dr}{d\theta} = \frac{-r^2}{b}\sqrt{1 \pm a^2/r^2}$$

$$u = 1/r$$

$$\frac{du}{d\theta} = \frac{1}{b}\sqrt{1 \pm a^2u^2}$$

$$\theta = b\int_0^u \frac{du}{\sqrt{1 \pm a^2u^2}} = \frac{b}{a}\sinh^{-1} a/r$$

for $b^2 < V_0/E$

For $\ominus$ sign, answer becomes

$$\theta = b\sin^{-1} a/r, \quad \text{for } b^2 > V_0/E$$

if $a^2 = 0$

$$\theta = b\int_b^u du = \frac{b}{r}$$

$$r = \begin{cases} a/\sinh\left(\frac{a}{b}\theta\right), & b^2 < V_0/E, \quad a = \sqrt{\frac{V_0}{E} - b^2} \\ a/\sin\left(\frac{a}{b}\theta\right), & b^2 > V_0/E, \quad a = \sqrt{b^2 - V_0/E} \\ b/\theta, & b^2 = V_0/E \end{cases}$$