your name(s) KEY

Physics 321 Quiz #1 - Friday, Jan. 11th

Work in groups of 3 (assigned in class) to complete this assignment.

1. Using the definition and anti-symmetrization properties of the Levi-Civita symbol defined in Eq. (1.16) from the lecture notes,

   (a) (5 pts) show that

   $$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}.$$

   $$\varepsilon_{ijk} A_j B_k = -\varepsilon_{ikj} A_j B_k = -\varepsilon_{ikj} B_k A_j = -(\vec{B}\times\vec{A})_i$$

   (b) (5 pts) Express

   $$\vec{A}\times(\vec{B}\times\vec{C}) - (\vec{A}\times\vec{B})\times\vec{C}$$

   without any vector products (but you can use dot products). Hint: Use Eq. (1.17) from the lecture notes.

   $$= \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) + \vec{C}\times(\vec{A}\times\vec{B})$$

   $$= \vec{B}(\cancel{\vec{A}}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) + \vec{A}(\vec{B}\cdot\vec{C}) - \vec{B}(\cancel{\vec{A}}\cdot\vec{C}) = \vec{A}(\vec{B}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

   (c) (5 pts) Express

   $$\nabla\times(\nabla\times\vec{A}) - \cancel{(\nabla\times\nabla)\times\vec{A}}$$

   without any curl operators.

   $$= \nabla(\vec{\nabla}\cdot\vec{A}) - (\nabla\cdot\nabla)\vec{A}$$

   $$= \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A$$

2. Consider two velocities $\vec{a}$ and $\vec{b}$.

   (a) (5pts) Find a vector $\vec{a}\,'$ that is parallel to $\vec{b}$, and with a magnitude such that

   $$\vec{a}\,'\cdot\hat{b} = \vec{a}\cdot\hat{b},$$

   i.e. $\vec{a}\,'$ is the same as the component of $\vec{a}$ in the $\hat{b}$ direction if one were to use a coordinate system where the vector $\vec{b}$ was along one of the coordinate axes defined by a unit vector $\hat{b}$. One can state these constraints as $\vec{a}\,'\times\vec{b} = \mathbf{0}$ and $\vec{a}\cdot\vec{b} = \vec{a}\,'\cdot\vec{b}$. Express you answer using dot products.

   $$\vec{a}\,' = \hat{b}(\hat{b}\cdot\vec{a}) = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\, b$$

   (b) (5 pts) Repeat the same, but find a vector $\vec{a}\,'$ in a coordinate system where $\hat{b}$ is one of the defining unit vectors, such that the two components of $\vec{a}\,'$ perpendicular to $\vec{b}$ are the same as for $\vec{a}$, but the component of $\vec{a}\,'$ in the direction of $\hat{b}$ is zero. One can state this as $\vec{a}\,'\cdot\vec{b} = \mathbf{0}$.

   $$\vec{a}\,' = \vec{a} - \frac{(\vec{a}\cdot\vec{b})\vec{b}}{\vec{b}\cdot\vec{b}}$$